MAG01: Magnetic Levitation (MAGLEV)

Interdisciplinary Automatic Controls Laboratory - ME/ECE/CHE 389

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1 Introduction

1.1 Overview and goals

The Magnetic Levitation System, MagLev for short, is inherently nonlinear and open loop unstable. Maglev trains and magnetic bearings are two of the most important related applications. In this project we consider a single degree of freedom magnetic suspension system. The experiment will explore issues associated with nonlinearity, instability, and robustness of control design to modeling errors.

1.2 Apparatus

The levitation system consists of a position sensor, actuator, and controller. Figure 1.2 shows a conceptual schematic of this system. The actuator provides the force necessary to counteract gravity and to stabilize the equilibrium. It is an electromagnet whose field strength depends on the amount of current flowing in the coil. We can thus control the magnetic force by adjusting this current. Fortunately, the equilibrium is passively stabilized in the lateral directions via the field gradient, so we only need to consider control over the vertical displacement. A suspension system also requires a mechanism for sensing the position of the object. Here, we use an optical approach with a IR light source and associated sensor. As the ball moves

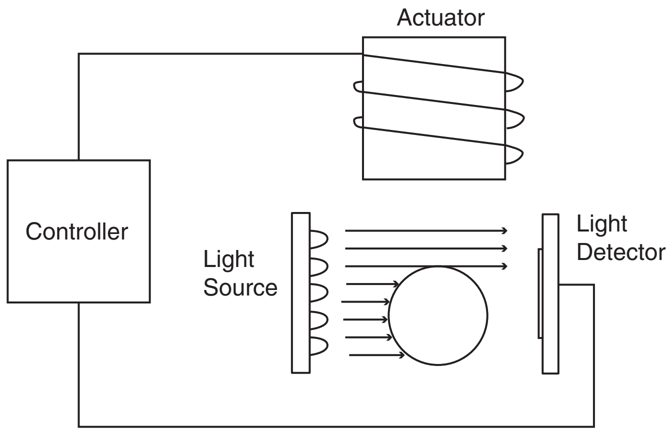


Figure 1: System schematic consisting of actuator position sensor and controller.

up and down, the amount of light detected changes accordingly. The controller looks at the position of the ball and compares it to the reference position, adjusting the current as needed to suspend the ball.

1.3 Matlab Files

The relevant files are located on the C drive.

- `C:\33-942 Magnetic Levitation`
- Run the file `startup.m` to set the matlab path
- Find a Maglev Simulation code on the controls lab drive `MagLevSimulation.m`

2 Modeling

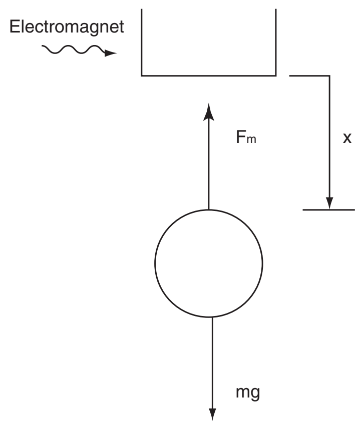
There are two forces acting on the ball, the gravitational force, mg, and the magnetic force, which is given by

$$F_m = k\frac{i^2}{x^2} \tag{1}$$

where x is gap spacing between the pole of the electromagnet and the ball, i is the current through the inductor, and k is an electromagnetic force constant, which is dependent on the material properties and physical structure of the magnet. The magnetic force relation is a simplified approximation for the system; it ignores many non-ideal characteristics. Specifically, the equation does not account for effects including finite core reluctance, saturation of the core, magnetic hysteresis, and eddy currents in the core.

Assuming the downward direction represents positive displacement, the dynamics of the system can be described by

$$m\ddot{x} = mg - k\frac{i^2}{x^2}. \tag{2}$$



Variable	Description	Units	Value
x	Vertical displacement	(m)	
F_m	Magnetic force exerted on ball	(N)	
m	Mass of ball	(kg)	0.0218
k	Constant	(Nm^2/A^2	2.48×10^{-5}
i_0	Equilibrium Current	(A)	0.8
x_0	Equilibrium Position	(m)	0.009
g	Gravitational acceleration	(N/m^2)	9.81

Figure 2: Free body diagram of maglev ball.

Lab Work 1: State Space System Modeling

This system (2) can be written in an equivalent state space form, which is useful for simulation purposes. Since this is a second order system, it requires two state variables to completely define the dynamics. Define the state variables $x_1 = x$ and $x_2 = \dot{x}$, and construct the state space system

2.1 Model Linearization

The suspension of a ball with an electromagnet is difficult because it is open-loop unstable and there is a nonlinear relationship between force, current, and gap between the pole of the electromagnet and ball. Equilibrium is reached when the magnetic force balances the gravitational force. Intuitively it makes sense that the system is unstable. Imagine the ball is sitting at the equilibrium point under a fixed magnetic field, a small deviation towards the magnet will increase the magnetic force perturbation or a small deviation away from the magnetic will decrease the magnetic force allowing the ball to fall, also growing the perturbation. One effective method to stabilize a nonlinear system around an operating point is to take the first order approximation of the system around that operating point, a linearization, and then proceed with standard control techniques for linear systems.

Let x_0 and i_0 represent some equilibrium point, i.e.

$$\ddot{x}_0 = 0, \quad \text{and} \quad mg = k\frac{i_0^2}{x_0^2}. \tag{3}$$

Define the new variables, $\bar{x}$ and $\bar{i}$ as perturbations around the equilibrium x_0 and i_0, i.e.

$$\begin{aligned} x &= x_0 + \bar{x}, \\ i &= i_0 + \bar{i}. \end{aligned} \tag{4}$$

For small perturbations around the equilibrium, the nonlinear system (2) can be well described by the first order approximation,

$$\begin{aligned} F_m &\approx k\frac{{i_0}^2}{{x_0}^2} + \left.\frac{\partial F_m}{\partial x}\right|_{i_0,x_0} \bar{x} + \left.\frac{\partial F_m}{\partial i}\right|_{i_0,x_0} \bar{i}, \\ F_m &\approx k\frac{{i_0}^2}{{x_0}^2} + -2k\left(\frac{i_0^2}{x_0^3}\right)\bar{x} + 2k\left(\frac{i_0}{x_0^2}\right)\bar{i}. \end{aligned} \tag{5}$$

With this approximation, we have a linear system which represents the dynamics well for small perturbations around the equilibrium,

$$m\ddot{\bar{x}} - k_1\bar{x} = -k_2\bar{i} \tag{6}$$

where $k_1 = 2k\frac{i_0^2}{x_0^3}$ and $k_2 = 2k\frac{i_0}{x_0^2}$.

For $k_1 > 0$, the poles of the system are at

$$s_1 = -\sqrt{\frac{k_1}{m}}, \quad s_2 = \sqrt{\frac{k_1}{m}}. \tag{7}$$

Notice there is a right half plane pole, which represents the open-loop instability of the system. Clearly, compensation efforts have to focus on moving the right-half plane pole into the stable left-half plane.

Lab Work 2:

1. Construct a pole zero map and bode plot of the open loop system for varying values of k_1 (2). k_1 can be used to describe uncertainty in the system, so, we should understand how the dynamics change with varying values of k_1.

2. For the value of k_1 provided in Figure 2, plot the position of the closed-loop poles with a proportional controller. Is there anyway to stabilize this system with proportional control?

3. What is the simplest controller that could be used to stabilize the system? How can we pull the closed-loop poles into the LHP? Note, that we don't really know the value of k_1 exactly, and there are other unmodeled dynamics, so there must be some consideration for uncertainty.

3 Maglev Control

A lead compensator along with sufficiently high gain will pull the pole over into the left-half plane. The drawback of this approach is that it is only valid for small deviations around an operating point. It is however, simple and straightforward to implement, and leads to natural representation in state-space form, which allows easy computation.

3.1 Lead Control

A lead compensator has the form

$$G_c(s) = K(\alpha\tau s + 1), \tag{8}$$

which, you can note, is just a PD controller. The uncompensated plant has zero phase margin (phase is 180 deg for all frequencies). Lead compensation can be used to add phase in the neighborhood of the crossover frequency. A more practical lead compensator has the form

$$G_c(s) = K\frac{\alpha\tau s + 1}{\tau s + 1}. \tag{9}$$

The pole is added to make the system strictly proper. Otherwise the gain would be unrealizable at high frequencies, and would amplify noise. The network has a low frequency zero followed by a higher frequency pole. Therefore, when the phase from the zero starts to take effect the magnitude has not yet begun to rise significantly, and it is possible to leave the crossover frequency unchanged. Typically, α is set to about 10, i.e. the pole-zero pair is a decade apart in frequency, and the crossover frequency is placed at the geometric mean of the pole-zero pair for maximum phase improvement.

Lab Work 3: Lead Control Design via Root Locus Analysis

Use root-locus design techniques to design a lead compensator which stabilizes the linearized system

1. Design a lead controller by using the zero to cancel out the LHP pole, then the lead pole or controller gain can be chosen to satisfy design requirements. Design a controller which satisfies a damping coefficient of the $\zeta = 0.7$.

2. Why might this controller design not be suitable for the actual physical system. How can we design the controller to ensure it still works for a the physical system. Think about model uncertainty.

The maximum phase is given by

$$\phi_m = \sin^{-1} \frac{\alpha - 1}{\alpha + 1} \tag{10}$$

The lead compensator will move the poles into the left-half plane but we must properly choose the parameters to achieve desired performance.

Lab Work 4: Lead Control Design via Bode Analysis

Use bode design techniques to design a lead compensator which stabilizes the linearized system

1. Use bode analysis to show that a PD controller can be well approximated by a practical lead controller.

2. Use the bode plot of the lead controller to show why a lead controller "adds phase" to the system.

3. Design a Lead compensator to achieve a phase margin of 30°and a cross over frequency which satisfies speed of response requirement specified by the setting time.

   - $t_s < 1$
   - PM $> 30°$

4. Draw the bode plot of the compensated system to show satisfaction of the PM at the desired cross over frequency.

5. Draw the root locus of the compensated system, how does this compare the previous control design via root-locus techniques.

6. You can test the lead compensator with the matlab simulation file `MagLevSimulation.m`. Or build your own simulation. It may be easier to start with a very simple simulation using built in matlab functions `tf` and `lsim`. Then use the simulation file to add more complexity to the simulation such as measurement noise and for generating nicer plots.

4 Maglev Modeling: Inclusion of circuit dynamics

A Model Identification

$$T = \frac{G}{1 + CG} \tag{11}$$

With lead compensator of the form

$$C = K\frac{z + z_c}{z + p_c} \tag{12}$$

where $K =$, $z_c =$, and $p_c =$. written in terms of the model parameters the closed loop transfer function is

$$T = \frac{2000k_1 + 4439k_1 z^{-1} + 2878k_1 z^{-2} + 439k_1 z^{-3}}{(155120k_1 + 200k_2 - 8 \times 10^9) + (57560k_1)z^{-1} + ()z^{-2} + ()z^{-3}} \tag{13}$$