ME 433 – STATE SPACE CONTROL

Lecture 6

State Observer

Problem Definition: "An unforced system is said to be observable if and only if it is possible to determine any (arbitrary initial) state $x(0)$ by using only a finite record, $y(\tau)$ for $0 \le \tau \le T$, of the output"

Theorem: "A system is observable if and only if the matrix

$$\bar{O} = \begin{bmatrix} C \\ CA \\ \vdots \\ CA^{n-1} \end{bmatrix}$$

Observability Matrix

is full-rank."

State Observer Design

We consider the linear, time-invariant system

$$\dot{x} = Ax + Bu,$$
$$y = Cx + Du.$$

and we look for an "observer" of the state of the form

$$\dot{\hat{x}} = A\hat{x} + Bu + L(y - \hat{y}),$$
$$\hat{y} = C\hat{x} + Du.$$

In this case we have the error $\tilde{x} = x - \hat{x}$ dynamics

$$\dot{\tilde{x}} = (A - LC)\tilde{x}$$

We should note that we can modify the dynamics (eigenvalues) of the error system by proper selection of the gain L. If the system is observable, it is always possible to find an observer gain L to set the eigenvalues of the error dynamics at arbitrary values.

State Observer Design

By noting that

$$eig\{A - LC\} = eig\left\{(A - LC)^T\right\} = eig\left\{A^T - C^T L^T\right\}$$

we can conclude that the observer eigenvalue placement problem is similar to the controller eigenvalue placement problem

$$eig\{A - BK\}$$

By making

$$A = A^T, B = C^T$$

we can use the same eigenvalue placement formulas developed by state feedback control design. After obtaining K, we obtain L as

$$L = K^T$$

State Observer Design

Examples :

Reduced State Observer Design

We consider the linear, time-invariant system

$$\dot{x} = Ax + Bu,$$
$$y = Cx + Du.$$

Let us assume that p of the n states can be measured. Let us partition the state vector as

$$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

where $x_1 \in R^p$, and $x_2 \in R^{n-p}$. The system dynamics can be written as

$$\dot{x}_1 = A_{11}x_1 + A_{12}x_2 + B_1u,$$
$$\dot{x}_2 = A_{21}x_1 + A_{22}x_2 + B_2u.$$

And the observation of the system is given by

$$y = C_1x_1 \Rightarrow x_1 = C_1^{-1}y$$

Reduced State Observer Design

Since $x_1 \in R^p$ is measurable, we only need to estimate $x_2 \in R^{n-p}$. We propose

$$\hat{x}_2 = Ly + z$$

where

$$\dot{z} = Fz + Gy + Hu.$$

If we choose

$$F = A_{22} - LC_1A_{12}$$
$$H = B_2 - LC_1B_1$$
$$GC_1 = A_{21} - LC_1A_{11} + FLC_1$$

the dynamics of the estimation error is governed by

$$\dot{e}_2 = Fe_2, \qquad e_2 = x_2 - \hat{x}_2$$

Proof: In class.

Output Feedback

Separation Principle:

1. Design the control law under the assumption that all state variables in the process can be measured.
2. Design an observer to estimate the state of the process for which the control law of step 1 was designed.
3. Combine the full-state control law design of step 1 with the observer design of step 2 to obtain the compensator design.

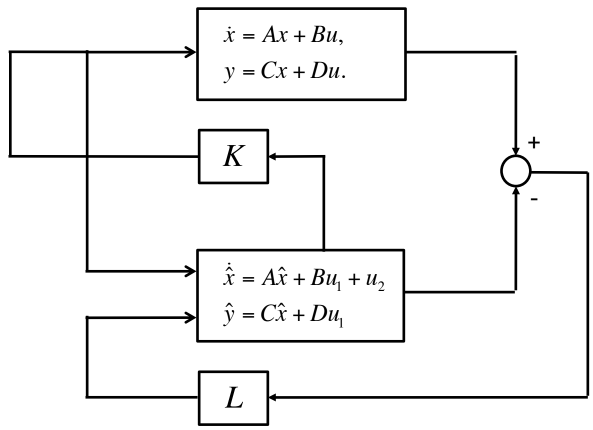
In other words,

$$u = -K\hat{x}$$

Output Feedback

We consider now the following feedback law

$$u = -K\hat{x}$$



Output Feedback

System

$$\dot{x} = Ax - BK\hat{x}$$
$$y = Cx - DK\hat{x}$$

Observer

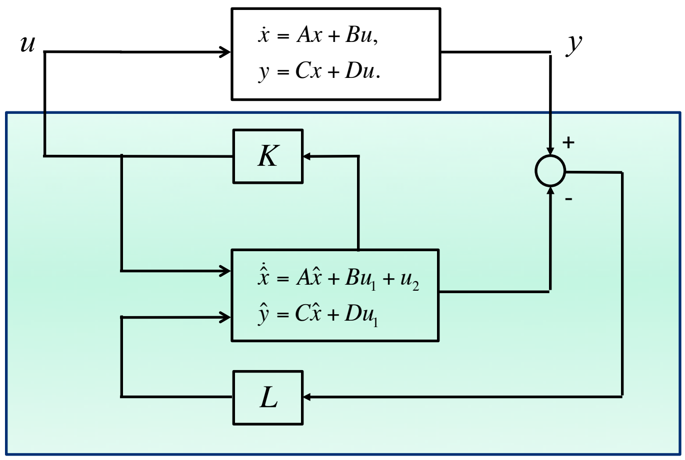
$$\dot{\hat{x}} = (A - LC - BK)\hat{x} + LCx$$
$$\hat{y} = (C - DK)\hat{x}$$

$$\begin{bmatrix} \dot{x} \\ \dot{\hat{x}} \end{bmatrix} = \begin{bmatrix} A & -BK \\ LC & (A - LC - BK) \end{bmatrix} \begin{bmatrix} x \\ \hat{x} \end{bmatrix}$$
$$\begin{bmatrix} y \\ \hat{y} \end{bmatrix} = \begin{bmatrix} C & -DK \\ 0 & C - DK \end{bmatrix} \begin{bmatrix} x \\ \hat{x} \end{bmatrix}$$

Output Feedback

We consider now the following feedback law

$$u = -K\hat{x}$$



Output Feedback

State Feedback

$$u = -K\hat{x}$$

Observer

$$\dot{\hat{x}} = (A - LC - BK)\hat{x} + Ly$$
$$\hat{y} = (C - DK)\hat{x}$$

Output Feedback

$$\dot{\hat{x}} = (A - LC - BK)\hat{x} + Ly \qquad A_{cont} = (A - LC - BK), B_{cont} = L$$
$$u = -K\hat{x} \qquad C_{cont} = -K, D_{cont} = 0$$

$$G_{cont}(s) = \frac{U(s)}{Y(s)} = -K\left[sI - (A - LC - BK)\right]^{-1} L$$

Stabilizability and Detectability

If the uncontrollable part of a system is stable, we say that the system is ***stabilizable***. An the uncontrolled part often can be ignored by the control designer.

If the unobservable part of a system is stable, we say that the system is ***detectable***. An the unobservable part often can be ignored by the control designer.