Relating Demand Behavior and Production Policies in the Manufacturing Supply Chain

S. David Wu
Lehigh University, Bethlehem, Pennsylvania
Mary J. Meixell
The Bell Laboratories, Lucent Technologies, Princeton, New Jersey

Abstract

Production decisions in a manufacturing supply chain are no longer driven by manual systems based on instinct and experience. They are regulated interactions between analysts, production managers and their collective manipulation of policies within the production information system. This paper studies the demand behavior in manufacturing supply chains and its relationships to the logic of production information systems pertaining to order batching, multiple schedule releases, bill of material processing, and resource leveling. The analysis helps us to understand the possible causes of demand amplification, the dependencies and mutual sensitivities of production decisions over multiple supply tiers, and the possible effects of production lot sizes, product design, and capacity levels. Main results include that multiple schedule releases may be used to stabilize supply chain operations, production batching may amplify demand variations across supply tiers, and that operating close to capacity tends to dampen demand variation downstream.

Subject Categorizations:
Production/Scheduling, planning: manufacturing supply chain operations
Manufacturing, performance/productivity: demand behavior analysis

1. Introduction

Demand variation is a major source of uncertainty in manufacturing supply chains. A lack of understanding in demand processing may lead to unjustifiably high inventory, excess capacity, or overly nervous operational policies; all translate into significant capital and costs. Leading manufacturers seek to improve the consistency of their supply chain demand information using EDI and integrated production information systems. After aggressive development over a decade, it is now common place in the automotive industry to use integrated order management systems to handle production/demand information across tiers of manufacturing plants and suppliers. In this environment, demand is communicated along the supply chain as production schedules posted for its immediate suppliers, i.e., a customer's schedule represents demand at that particular point in time. A supplier uses this information to generate its own production schedules, and then posts its schedule for its supply base to use. Sophisticated as they are, these information systems are not without problems. It is well known in the industry that these schedules change frequently and sometimes erratically. Many supplier failures can be traced to a customer schedule change that is unmanageable for that particular supplier at that particular time. On the other hand, customer schedule changes can often be traced to a supplier performance failure. This spiraling effect in the context of a massive production information system forms the complex structure of modern manufacturing supply chains.

This paper examines demand behaviors in a manufacturing supply chain by characterizing the basic decision logic of the production information system. A basic thesis of the paper is that the process driving the decisions in manufacturing supply is a regulated interaction between analysts, production managers and their collective manipulation of the production information system. By examining the operational logic of batching, multiple schedule releases, product structure, capacity utilization and its effects on demand propagation, we will be able to gain basic insights on supply chain demand behavior.

Research in supply chain demand analysis has attracted a lot of attention in recent years. The work by Lee et al. (1997), for instance, brings to the forefront of research the pathological demand behavior known as the bullwhip effect. Although the behavior is well known to the practitioners, its underlying processes were not well understood. This approach of studying basic supply chain phenomena under generalized setting (c.f. Sterman, (1989), Slats et al (1995), Lee and Billington (1992)) is important in that they provide managerial insights under rather mild assumptions. Once understood, a decision-maker can improve performance by isolating and avoiding the source of the problem. Another line of research concentrate on identifying policies to avoid the undesired supply chain behavior (Takahashi, et al, (1987); Towill, (1992), Lee, (1996)). To date, much of the research in supply chain management has been focusing on retail applications, transportation logistics, distribution channel design, or service part logistics. This paper will focus on the production pipelines in the manufacturing supply chain.

Multi-level lot sizing models and solution methodologies form the backdrop of modern manufacturing planning systems. These models loosely define the so-called “MRP logic.” The lot sizing models are studied intensively over the past 30 years. Various reviews of the literature are available (c.f., Bahl et al. (1987), Maes and van Wassenhove (1988), Nam and Logendran (1992), Kimms (1997)). Although most of these models are intended for facility-level production planning, they have been generalized in recent years to handle broader scopes of production planning (Bhatnagar et al. 1993, Thomas and Griffin (1996), Vidal and Goetschalckx (1997)). Several researchers focus on broader information technology aspects of supply chain implementation (Camm et al. (1997), Geoffrion and Power (1995), Gunasekaran and Nath (1997), and Singh (1996)). And many documented industry cases (Lee and Billington (1995), Martin et al. (1995), Robinson et al. (1993), Arntzen et al. (1995)) and supply chain implementation topics.

The focus of this paper is supply chain demand behavior. It differs from a majority of manufacturing planing research in that we are not interested in a *prescriptive* model that produces some optimized global solution that minimizes costs.

Such solution is unlikely to be adopted by the diverse manufacturing facilities in the supply chain and is therefore of no direct decision value. Rather, we are interested in understanding the behavior of systems operating *under* such manufacturing planning systems. Since most existing systems today operate under some enterprise resource planning (ERP) structure using different variations of the MRP logic, we believe it is possible to characterize at an aggregate level the operational behavior of manufacturing supply chains.

2. Manufacturing Supply Chain Structure

A manufacturing supply chain is a network of facilities $k=1,\ldots,K$ and processes needed to manufacture a variety of products, $i=1,\ldots,N$. Flow in a supply chain is bi-directional. Information concerning demand and production planning flows down, while material and subassemblies to support upper tier manufacturing processes flow up. The final product for the supply chain is represented as the top tier in the chain, and customer sales for that item make up the demand. There can be multiple tiers of "intermediate products" or *components* in the network, organized by the number of levels each is removed from the final product at the top tier. The first tier suppliers ship directly to final assembly, the second tier ships to first tier, and so on. Each product in each tier has its own set of components, following the form of its bill-of-material (BOM) structure. Product structure is a network characterized by matrix $[a_{ji}]$, where a_{ji} represents the number of units of component j required for the production of one unit of item i. One or more BOM structures are embedded in the manufacturing supply chain structure where a facility k supplies to multiple customers and is served by multiple suppliers. Components I_k made within the facility is a collection of nodes from multiple product structures. A component j may be shared by multiple products i in several different BOM structures. Thus, the planned production x_{jt} for an upper tier product i imposes a demand $(a_{ji}x_{it})$ on its lower tier components j.

2.1 Demand Dependency in the Supply Chain

One central characteristic of supply chains is the dependency in demand throughout the chain. Demand for end items is an input to the scheduling process at the final manufacturing tier, and supplier schedules are generated to meet the schedule requirements. These supplier orders then comprise the demand for products at the next tier. This "orders to schedule to requirements" process is common in manufacturing supply chains. As mentioned earlier, in the automotive industry, "just-in-time delivery" is the common practice where suppliers generate their production schedules based on their customer's schedule posted electronically. A delivery is made strictly based on the customer's demand as described in the schedule, i.e., a planned production x_{it} for product i triggers a series of orders with size $(a_{ji}x_{it})$, to the designated supplier of each component j. Although a two to three week inventory buffer is normally kept, the dependency on demand changes is significant. This sensitivity to demand changes underlines the importance of planning coordinating within the chain.

Under the above operational assumptions, schedules along a supply chain are dependent on the previous tier. However, the perceived internal demands within a manufacturing supply chain vary along the chain. Overall demand perceived at tier k-1 might differ from those at tier k, which differ from those at tier $k+1$. This variation exists even though demand in a supply network is dependent on the next higher tier because schedules are constantly changing and progressively modified as they pass through each tier in a supply chain. In reality, the perceived demand for item i at time epoch τ is determined from the announced production x^{τ}_{jt} for product j at an upper tier facility, with "proper adjustment" by local production planning staff. This results in a nonnegative order size of $(a_{ij}x^{\tau}_{jt}+\rho^{+}_{jt}-\rho^{-}_{jt})^{+}$ where ρ^{+}_{jt} represents a positive adjustment (padding) for item j in period t and ρ^{-}_{jt} represents a negative adjustment (shrinking). Consider further the existence of an external demand r_{it} for item i in period t, a facility at a particular time epoch τ will try to satisfy the following equation for each of its products in each foreseeable future periods. We assume that a product is held in inventory only at the facility that produces it. Let y_{it} denote the inventory of product i at the end of period t and L_i the lead time required to produce item i. Thus we have the following equality:

$$y_{i,t-1} + f_i x_{i,t-L_i} - y_{it} = \sum_{j=1}^{N} (a_{ij} x_{jt}^{t} + r_{jt}^{+} - r_{jt}^{-}) + r_{it}, \qquad \forall i,t \qquad (1)$$

Note this equation takes the form of a well-known inventory balance equation, which maintains the dynamic Leonteif structure (Veinott 1969). It states that the internal and the external demands for item i in period t (the right hand side terms) are satisfied by inventory carried over from the end of period t-1 and the production planned in period t-L_i, minus the inventory to be carried at the end of period t. In traditional analysis, the terms related to adjustment r^+ and r^- are not present, assuming no adjustment to the announced production orders.

2.2 Demand Amplifications in the Supply Chain

We next define in this section the notion of demand amplification. Numerous effects influence the pattern of which demand propagates through the supply chain. Consider an item i and its component j. The production lot-size established for item i imposes a consolidated demand quantity for its component j. The policy under which these lot-sizes are determined at each tier determines, to a large extend, the demand behavior in a supply chain. We will define three types of demand amplification in supply chains. The first is the degree to which demand varies from an item to its immediate component within a single schedule release.

Definition 1. (Item-Component Demand Amplification- Single Release, DA^s_{ij}) *For an item i and its immediate component j, item-component demand amplification,* $\boldsymbol{DA^s_{ij}}$, *is the change in coefficient of variation between item i demand and component j demand in a single schedule release. Specifically,*

$$DA^s_{ij} = CV(X) - CV(D) \qquad (2)$$

where random variable D represents the demand of item i over periods $t=L_i+1,...,T$, and random variable X represents non-zero internal demand generated from D over periods $t=1,...,T-L_i$, using some lot sizing policy.

Thus, DA^s measures the demand amplification between an item and its component within one single schedule release. Since a schedule release covers multiple time periods, DA^s pertains to multiple periods in the planning horizon. Suppose a leveled schedule is used in the system then the demand would be a constant quantity and *CV's* will be 0. Any other policy carries some level of variation.

In a manufacturing supply chain the production schedule is updated frequently via multiple schedule releases. In the following, we define a second type of demand amplification applies to this multi-release environment.

Definition 2. (Item-Component Demand Amplification- Multiple Releases, DA^m_{ij}) *For an item i and its immediate component j, item-component demand amplification,* ***DA^m_{ij},*** *is the change in coefficient of variation between item i demand and component j demand in multiple schedule releases. Specifically,*

$$DA^m_{ij} = CV(X^m) - CV(D^m) \qquad (3)$$

where random variable D^m represents the demand of item i over periods $t=L_i+1,...,T$, in schedule releases $t=1,..,Q$, and random variable X^m represents non-zero internal demand generated from D over periods $t=1,...,T-L_i$,in corresponding releases $t=1,..,Q$, using some lot sizing policy.

Thus, DA^m measures the demand amplification between an item and its component over multiple schedule release. Similar to DA^s, DA^m pertains to multiple periods in the planning horizon. A third type of demand amplification measures the changes in demand variation over two supply tiers. Consider a segment of a supply chain that is comprised of two supply tiers *k* and *l* where some items *i's* are produced in tier *k* and some of *i's* components are produced in tier *l*.

Definition 3. (Tier-to-Tier Demand Amplification, TA_{kl}) *For supply tiers k and l, tier-to-tier demand amplification* ***TA_{kl},*** *is the change in coefficient of variation between tier k and tier l in a single schedule release. Specifically,*

$$TA_{kl} = CV(X^l) - CV(D^k) \qquad (4)$$

where random variable D^k represents the demand of all items at tier k over all periods $t=1,...,T$, and random variable X^l represents the demand of all items at tier l over all periods $t=1,...,T$.

Using the basic manufacturing supply chain structure and the notion of demand amplification we will now explore several analytical insights concerning demand behavior.

In (Meixell and Wu, 1998), we conduct computational testing based on these theoretical results which provide empirical insights on more complex cases.

2.3 The Leontief Structure

The Leontief structure is an important structural property for mathematical models minimizing a concave function over the solution set of a Leontief substitution system. Lot sizing models are well known to belong to this class of problems (Veinott, 1969, Shapiro, 1993). A matrix is Leontief if it has exactly one positive element in each column and there is a nonnegative (column) vector x for which Ax is positive (i.e., has all positive components). Consider a supply chain production model using the objective function of the following form:

$$v = \min \sum_{i=1}^{N} \sum_{t=1}^{T} (h_i y_{it} + C_{it} x_{it})$$

where

$$C_{it} = \begin{cases} c_{it} x_{it} + K_i & if\ x_{it} > 0 \\ 0 & if\ x_{it} = 0 \end{cases} \qquad (5)$$

subject to the inventory balance constraints (1) and any other linear constraints. We have the following results from the Leoteif Structure.

Lemma 2.1. *The inventory balance constraint (1)*

$$y_{i,t-1} + f_i x_{i,t-L_i} - y_{it} - \sum_{j=1}^{N} (a_{ij} x_{jt} + r_{jt}^{+} - r_{jt}^{-}) = r_{it} \quad , \forall i, t$$

is a Leontief substitution system so long as the gross adjustment made through the supply tiers create nonnegative internal demands, i.e., $r_{it} + \sum_{j=1}^{N} (r_{jt}^{+} - r_{jt}^{-}) \geq 0$ *and*

$$\sum_{j=1}^{N} (a_{ij} x_{jt} + r_{jt}^{+} - r_{jt}^{-}) \geq 0 . \quad ð$$

The above result can be shown by rewriting equation (1) as follows:

$$y_{i,t-1} + f_i x_{i,t-L_i} - y_{it} - \sum_{j=1}^{N} a_{ij} x_{jt} = r_{it} + \sum_{j=1}^{N} (r_{jt}^{+} - r_{jt}^{-}) \quad ,\forall i,t$$

With known tier-to-tier adjustment r the right hand side is just a non-negative constant. Thus, the matrix of coefficients is Leonteif according to the results in Veinott (1969). As argued in Veinott (1968), the optimal solution for such a system is an extreme point solution. Furthermore, if one variable appears with a positive coefficient in the same balance constraint, only one of the variables can be positive in the optimal solution. Thus, from Lemma 2.1 we have the following application of Theorem 6 of Veinott (1968).

Corollary 2.1 *If equation (1) holds, and internal demands after adjustment is nonnegative, no production of item i will be scheduled if there is inventory carried over from a previous period, i.e.,* $y_{i,t-1} \cdot x_{i,t-L_i} = 0$

This result stipulates that either there is inventory for item i entering period t, ($y_{i,t-1}>0$) or there has been production initiated in period $t-L_i$ ($x_{i,t-Li}>0$), but not both. A system would not incur an additional setup for the same demand quantity if sufficient capacity was available to build it in a period that already incurred a setup. So, as capacity is added to a system, and production in the desired period could be incurred, $x_{i,t-Li}$, no inventory would be held for production in that period.

3. The Impact of Setup/Inventory Cost Ratio on Demand Amplification

3.1 The Effect on Item-Component Demand Amplification in a Single Release

We now consider the effect of setup/inventory cost ratio on demand amplification. Without loss of generality, we will first consider the case where capacity is non-binding. We will later provide a separate analysis for the capacity-restricted cases. Consider the following proposition:

Proposition 1. *When the setup/inventory cost ratio* $d/h \to 0$ *for item i and its component j (i.e., when the inventory costs always dominate setup) and capacity is not binding, there will be no item-component demand amplification, i.e.,* $DA^{s}_{ij} = 0$.

Proof: From *Corollary 2.1* the Leontief property states that $x_{t-Li} \times y_{t-1}=0$. When capacity is not biding and the inventory cost dominates the setup cost, a facility will build each order as needed and carry no extra inventory ($x_{t-Li}>0$ and $y_{t-1}=0$). As a result, the demand for the end item, r_{it}, translates through the supply chain (based on the product structure matrix $[a_{ij}]$) un-altered by production scheduling, and becomes the internal demand for a component, $x_{j,t-Li}$. Denote $x_{j1}, x_{j2},\ldots,x_{jT}$, the occurrences of random variables $X_1,\ldots,X_T$ *(internal component demands)*, and r_{i1}, $r_{i2},\ldots,r_T$ the occurrences of random variables $D_1,\ldots,D_T$ *(external item demands)*, respectively

Thus,

$E(X) = a_{ji}\, E(D)$ *and* $Var(X) = a^2_{ji}\, Var(D)$, therefore

$$
\begin{aligned}
&DA^s_{ij} = CV(X) - CV(D) \\
&= \frac{\sqrt{a^2_{ji} \cdot Var(D)}}{a_{ji} \cdot E(D)} - \frac{\sqrt{Var(D)}}{E(D)} = 0
\end{aligned}
$$

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Proposition 1 states that when the inventory cost dominates, the supply chain will build order as needed and there will be no item-component demand amplification in a single schedule release. On the other hand, when the inventory cost does not dominates the setup all the time the Leontief property ($x_{t-Li} \times y_{t-1}=0$) from *Corollary 2.1* implies that the supply chain will consolidate orders and build ahead, ($y_{t-1}>0$ and $x_{t-Li}=0$). Suppose v is the number of periods of orders are built ahead. This quantity v is bounded by u, the total number of periods with non-zero demand. Exactly how many periods of orders the facility will consolidate into one batch vary depending on the cost ratio d/h, but when $d/h>1$ the number of consolidated periods v ranges from 2 to u.

We will first start with the following proposition where the demands for item i are independent.

Proposition 2. *Suppose (as a result of order batching) v periods of orders for component j are combined to satisfy the internal demands imposed by item i, i.e.,* $x_{j,t-Li} = a_{ji}\,(r_{it} + r_{i,t+1} + \ldots + r_{i,t+v-1})$*, for* $t=L_i+1,\ldots,T-L_i$*. If the demand stream* $r_{i1},\ldots,r_{iT}$ *are drawn from iid random variables then the following are true after batching:*
(1) the mean and variance of component j's internal demand will increase by a factor of $a_{ji}\, v$ *and* $a^2_{ji}\, v$*, respectively*

(2) the item-component demand amplification DA^s_{ij} *decreases by a factor of* $(\frac{1}{\sqrt{v}}-1)$

Proof. Denote $x_{j1}, x_{j,v+1},\ldots,x_{j,T-v+1,}$ the realization of random variables $X_1,\ldots,X_{T-v+1,}$ and $r_{i1,}\, r_{i2},\ldots,r_T$ that of random variables $D_1,\ldots,D_T$, respectively, where $x_{j,t-Li} = a_{ji}\,(r_{it} + r_{i,t+1} +\ldots+r_{i,t+v-1})$, *for* $t=L_i+1,\ldots,T-L_i$. $D_1, D_2,\ldots,D_T$ are iid random variables with expected value $E(D)$ and variance $Var(D)$. As a result of lot-sizing, v periods of orders for component j are combined to satisfy the internal demands imposed by item i , this results in a stream X_1, $X_{v+1},\ldots,X_{T-v+1}$ for non-zero production periods where $X_1=a_{ji}(D_1+D_2+\ldots+D_v)$, $X_{v+1}=a_{ji}(D_{v+1}+\ldots+D_{2v})$, …, $X_{T-v+1}= a_{ji}(D_{T-v+1+}\ldots+D_T)$. The distribution of X is the convolution of that of D_t*'s*. Since D_t*'s* are iid random variables, we have

$$E(X) = a_{ji}\, v\, E(D) \text{ and } Var(X) = a^2_{ji}\, v\, Var(D)$$

This confirms statement (1).

Now, the item-component demand amplification is

$$
\begin{aligned}
&DA^s_{ij}\\
&= CV(X) - CV(D)\\
&= \frac{\sqrt{a^2_{ji}\cdot v\cdot Var(D)}}{a_{ji}\cdot v\cdot E(D)} - \frac{\sqrt{Var(D)}}{E(D)}\\
&= (\frac{1}{\sqrt{v}}-1)\cdot CV(D)
\end{aligned}
$$

This confirms statement (2) ð

The above result suggests that if the demands are independent, increasing the batch size in any production period increase the mean and variance of internal demands at the component level. Since both the variance and the total volume increase by a factor of v, the item-component demand amplification reduces as the lot size increases. However, it is important to note that DA^s_{ij} is defined such that we compute the coefficient of variation only over the periods where component j has *non-zero production.* All periods with no production (for j) after batching are dropped from the calculation, i.e., random variable X is a simple summation of random variable D every v periods, and we are only computing the variance and the CV of X. In Section 6 we will discuss the more complicated multi-item cases where both the non-production *and* the production periods for component *j* are included in the analysis.

We now present the results when demands are correlated.

Corollary 3.1. *Suppose the demands $r_{i1},\ldots,r_{iT}$ are drawn from correlated random variables with a coefficient of correlation r and the same variance, then the following are true after batching:*

(1) item-component demand amplification DA^s_{ij} change by a factor of $\left(\sqrt{\frac{1+vr-r}{v}}-1\right)$

(2) when r=1, there is no item-component demand amplification, i.e., $DA^s_{ij}=0$

Proof. The proof is similar to that of *Proposition 2* except that now

$E(X) = a_{ji}\, v\, E(D)$ but

$Var(X) = Var\,(a_{ji}D_1 + a_{ji}D_2 + \ldots + a_{ji}D_v)$

$$= \sum_{t=1}^{v} a_{ji}^2 \cdot Var(D_t) + 2\sum_{1\le t<s\le v} a_{ji}^2 \cdot Cov(D_t, D_s)$$

if $D_1, D_2,\ldots,D_T$ have the same variance i.e., $Var(D_t)=Var(D)$, then $Cov(D_t, D_s)=r\,Var(D)$ *and*

$Var\,(X) = a_{ji}^2 \cdot v \cdot Var(D) + a_{ji}^2(v^2-v)\cdot r \cdot Var(D) = a_{ji}^2 v(1+vr-r)Var(D)$, we have

$$DA^s_{ij} = CV(X) - CV(D)$$
$$= \frac{\sqrt{a_{ji}^2 \cdot v(1+vr-r)\cdot Var(D)}}{a_{ji}\cdot v \cdot E(D)} - \frac{\sqrt{Var(D)}}{E(D)}$$
$$= \left(\frac{\sqrt{v(1+vr-r)}}{v} - 1\right)\cdot CV(D)$$

This confirms statement (1). Statements (2) follows by setting r=1. ð

It should be noted that when demands are negatively correlated, the measure DA^s becomes problomatic when $v \geq 2$.

3.2 The Effect on Item-Component Demand Amplification in Multiple Releases

Now, we will look at the impact of batching to demand amplification when the demand of an item is a random variable across a series of schedule releases t=1,…, Q over multiple production periods t=1,…,T. Consider a particular schedule release t, let

r^t_{it} be the demand of item i in period t, and $x^t_{j,t\text{-}Li}$ be the internal demand imposed on i's component j in period $t\text{-}L_{i.}$ Suppose, after order batching, v periods of orders for component j are combined to satisfy the demands of item i, i.e., ., $x^t_{j,t\text{-}Li} = a_{ji}\ (r^t_{it} + r^t_{i,t+1} + \ldots + r^t_{i,t+v})$. We first present the proposition for the independent case.

Proposition 3. *Suppose (as a result of batching) v periods of orders for component j are combined to satisfy the internal demands imposed by item i, i.e.,* $x^t_{j,t\text{-}Li} = a_{ji}\ (r^t_{it} + r^t_{i,t+1} + \ldots + r^t_{i,t+v})$, *for* $t=L_i+1,\ldots,T\text{-}L_i$ *and* $t=1,\ldots,Q$. *If the demands* $r^1_{i1,\ldots,}r^1_{iT},\ldots,r^Q_{i1},\ldots,r^Q_{iT}$ *are drawn from iid random variables then the following are true:*
(1) the mean and variance of component j's internal demand will increase by a factor of $a_{ji}\ v$ *and* $a^2_{ji}\, v$, *respectively*
(2) the item-component demand amplification DA^m_{ij} *will change by a factor of* $(\frac{1}{\sqrt{v}} - 1)$

Proof. Denote $x^1_{j1,},\ x^1_{j,v+1}\ldots,\ x^1_{j,T\text{-}v+1,\ldots,}x^Q_{j1,},\ldots,x^Q_{j,T\text{-}v+1}$ the realization of random variables $X^1_1,\ X^1_{v+1,}\ldots,X^1_{T\text{-}v+1},\ldots,X^Q_1,\ldots,X^Q_{T\text{-}v+1,}$ and $r^1_{i1,}\ldots,r^1_{iT},\ldots,r^Q_{i1},\ldots,r^Q_{iT}$ that of random variables $D^1_1,\ldots,D^1_T,\ldots,D^Q_1,\ldots,D^Q_T$, respectively, where $x_{j,t\text{-}Li} = a_{ji}\ (r_{it} + r_{i,t+1} + \ldots + r_{i,t+v\text{-}1})$, *for* $t=L_i+1,\ldots,T\text{-}L_i$. Suppose $D^1_1,\ldots,D^1_T,\ldots,D^Q_1,\ldots,D^Q_T$ are iid random variables with expected value $E(D)$ and variance $Var(D)$. Similar to *Proposition 2, w*e have

$E(X) = v\ E(D)$ and $Var(X) = v\ Var(D)$

Therefore

$$DA^m_{ijt} = CV(x^{k+1,t}_{j,t-L_i}, t = 1,\ldots,\Theta, t = 1,\ldots,T - L_i) - CV(r^{k,t}_{it}, t = 1,\ldots,\Theta, t = L_i + 1,\ldots,T)$$

$= CV(X)\ \text{-}CV(D)$

$$= (\frac{1}{\sqrt{v}} - 1)\cdot CV(D) \qquad ð$$

The above result suggests that when the multiple releases are mutually independent, item-component demand amplification behave the same way as in the single release cases. However, a more interesting issue arises in a multiple release environment since different policies are possible in using the multiple schedule releases. A common practice in the industry can be described as a "freeze-up-to" policy as follows: freeze the period-t schedule from the customer at the end of period $(t\text{-}k)$, use this schedule as the internal demand and determine the production for period t.

Similarly, the release in period *t-k+1* is used to determine period *t+1* production, and *T-k* is used for period *T*, etc., only one period from each schedule is actually used while the remaining periods are discarded. Common as it is, this may not be a good policy if schedules continue to change up till the very end of period *t-1*. We will pursue this argument more rigorously by introducing the following formalism. Consider each order release for item *i* as a vector-valued random variable as follows:

$$D^t = \begin{bmatrix} D_1^t \\ \vdots \\ D_T^t \end{bmatrix} \quad for\,t = 1,\ldots,\Theta$$

After batching *v* periods of orders for production, this order release (customer's production schedule) is translated into scheduling releases as follows:

$$X^t = \begin{bmatrix} D_1^t + \ldots + D_v^t \\ \cdots \\ D_{T-v}^t + \ldots + D_T^t \end{bmatrix} = \begin{bmatrix} X_1^t \\ \vdots \\ X_{T-v+1}^t \end{bmatrix} \quad for\,t = 1,\ldots,\Theta$$

Each vector-valued random variable X^t represent a schedule release. If we write all the X^t *'s* together over multiple releases, we have a matrix as follows:

$$M_{(T-v+1)\times\Theta} = \begin{bmatrix} X_1^1 & & \cdots & & X_1^\Theta \\ & X_{v+1}^2 & & & \\ \vdots & & X_{2v+1}^3 & & \\ & & & \ddots & \\ X_{T-v+1}^1 & & \cdots & & X_{T-v+1}^\Theta \end{bmatrix}$$

Suppose a release index t is always k periods ahead of the time index, i.e., $t=t+k$, then we may represent the schedule using the above "freeze-up-to" policy as the diagonal terms of matrix *M* which is a random vector as follows:

$$S = \begin{bmatrix} X_1^1 \\ X_{v+1}^2 \\ \vdots \\ X_{T-v+1}^\Theta \end{bmatrix} = \begin{bmatrix} S_1 \\ S_2 \\ \vdots \\ S_{T/v} \end{bmatrix}$$

Consider an alternative policy where instead of following the schedule of the most recent releases we follow the expected value of all the available releases. In other words, we compute from matrix *M* a vector

$$\overline{X} = \begin{bmatrix} \overline{X}_1 \\ \overline{X}_{v+1} \\ \vdots \\ \overline{X}_{T-v+1} \end{bmatrix} \text{ where}$$

$$\overline{X}_1 = X_1^1$$
$$\overline{X}_{v+1} = (X_{v+1}^1 + X_{v+1}^2 + \ldots + X_{v+1}^v)/v$$
$$\overline{X}_{2v+1} = (X_{2v+1}^1 + \ldots + X_{2v+1}^v + X_{2v+1}^{v+1} + \ldots + X_{2v+1}^{2v})/2v$$

$\ldots$
$$\overline{X}_{T-v+1} = (X_{T-v+1}^1 + \ldots + X_{T-v+1}^v + \ldots + X_{T-v+1}^{v+1} + \ldots + X_{T-v+1}^{T-v})/(T-v)$$

We will now present the following important result.

Proposition 4. *Given multiple schedule releases from an upper tier customer, represented by matrix M, if the freeze-up-to schedule represented by random vector S has a variance of Var(S). Then an alternative policy that uses the expected value of all up-to-date releases (as defined by vector* $\overline{X}$ *) will reduce the variance by a factor of v for all but the first period.*

Proof. For the freeze-up-to schedule, define the first period $X_1^1 = S_1$, and define the rest of the period by the vector-valued random variable $S = \begin{bmatrix} X_{v+1}^2 \\ X_{2v+1}^3 \\ \vdots \\ X_{T-v+1}^{\Theta} \end{bmatrix} = \begin{bmatrix} S_2 \\ S_3 \\ \vdots \\ S_{T/v} \end{bmatrix}$

The mean and variance of *S* are as follows:

$$E(S) = \begin{bmatrix} E(X_{v+1}^2) \\ E(X_{2v+1}^3) \\ \vdots \\ E(X_{T-v+1}^{\Theta}) \end{bmatrix} = E(\overline{X})$$

and *Var(S)* is a matrix $S_s = [s_{ij}]$ where each entry $s_{ij} = Cov(S_i, S_j)$

but, $Cov(S_i, S_j) = 0$ *for* $i \neq j$ and $Cov(S_i, S_j) = Var(S_i)$ *for* $i = j$

Therefore, S_s is a scalar matrix and $Var(S) = S_s = Var(X) \cdot I$

Similarly, the mean and variance of random vector $\overline{X}$ are as follows:

$E(\overline{X}) = E(S)$, for the very first period, $Var(\overline{X}_1) = Var(S_1)$, from the $(v+1)$th period on

$Var(\bar{X}) = \Sigma_X$

$$= \begin{bmatrix} Var(\bar{X}_{v+1}) \\ Var(\bar{X}_{2v+1}) \\ \vdots \\ Var(\bar{X}_{T-v+1}) \end{bmatrix}^T \cdot I = \begin{bmatrix} Var(X)/v \\ Var(X)/2v \\ \vdots \\ Var(X)/(T/v)v \end{bmatrix}^T \cdot I = \frac{Var(X)}{v} . \begin{bmatrix} 1 \\ 1/2 \\ \vdots \\ v/T \end{bmatrix}^T .I = \frac{Var(X)}{v} \cdot c \cdot I$$

Where c is a fixed vector. Therefore, $Var(\bar{X}) = Var(S) \cdot c/v$. ð

The above result is important since it suggests that the practice of following a "most recent" schedule release is subject to a much higher (v times) variance then one which compute the schedule from the expected values of up-to-date releases.

4. The Impact of Product Structure on Demand Amplification

Up to this point, we have been focusing our attention on the demand amplification for a particular item-component pair. In an assembly product structure (see Figure 1) where each component feeds exactly one item, it is quite easy to generalize this analysis to multiple tiers. In a general product structure (also in Figure1), however, a component may feed into multiple items. In other words, the demands of multiple upper tier items may have a joint effect on components in a lower tier. This particular issue is important since many manufacturing supply chains are going through steps to consolidate their product designs such that a higher percentage of common components may be shared among different products. While this makes intuitive sense, some further analysis is necessary to understand the influence of product structure on demand amplification. We will examine this effect in the context of order batching established earlier.

Using the same notations, we define the assembly structure as the case where the production of component j, say x_{jt} will be used to satisfy the demand for exactly one type of item i, i.e., $x_{j,t-Li} = a_{ji}\ (r_{it} + r_{i,t+1} + \ldots + r_{i,t+v-1})$, *for* $t = L_i+1, \ldots, T-L_i$ where x_{j1}, $x_{j,v+1}, \ldots, x_{j,T-v+1}$, are the realization of random variables $X_1, \ldots, X_{T-v+1}$, and r_{i1}, $r_{i2}, \ldots, r_T$ are that of random variables $D_1, \ldots, D_T$, respectively. We know from *Proposition 2* that if item demands $D_1, \ldots, D_T$, are iid, $E(X) = a_{ji}\ v\ E(D)$ and $Var(X) = a^2_{ji}\ v\ Var(D)$. In a general product structure, the production of component j, say x_{jt} will be used to satisfy the demand for multiple items $i_1, \ldots, i_n$, for each item i, in other words,

$$x_{j,t-L} = a_{ji_1}(r_{i_1t} + \ldots + r_{i_1,t+v_1-1}) + \ldots + a_{ji_n}(r_{i_nt} + \ldots + r_{i_n,t+v_n-1}) \text{ , } for\ t=L+1,\ldots,T-L,$$

For notational convenience, we assume a common lead time L for all items, and $v_1,\ldots,v_n$ are the lot sizes for items $i_1,\ldots,i_n$, respectively. We will now present the main result:

Proposition 5: *Supply chain C_1 has an assembly product structure and supply chain C_2 has a general structure, as defined above. Suppose in C_2 the production of each component j is used to satisfy the demand for n items $i_1,\ldots,i_n$,. Then the following are true when comparing C_1 and C_2's internal demands at the component level:*

(1) If item demands are iid random variables, the internal demand for a next-tier component j in C_2 has a variance n times larger than that of C_1

(2) if item demands are correlated random variables with coiefficient ρ and the same variance, the internal demand for a next-tier component j in C_2 has a variance $n(1+n\rho-\rho)$ times that of C_1's

Proof. We first distinguish the assembly and the general product structure as follows:

For component *j* and item *i* in an *assembly* structure, define

$x_{j,t-Li} = g_0$ where $g_0 = a_{ji}(r_{it} + r_{i,t+1} + \ldots + r_{i,t+v-1})$;

g_0 is the realization of random variable G_0.

For component *j* and items $i_1,\ldots,i_n$ in a *general* product structure, define

$z_{j,t-L} = g_1 + \ldots + g_n$ where $g_k = a_{ji_k}(r_{i_kt} + \ldots + r_{i_k,t+v_k-1})$, z_{j1}, $z_{j,v+1},\ldots,z_{j,T-v+1}$, are the realization of random variables $Z_1,\ldots,Z_{T-v+1}$ and $g_1,\ldots,g_n$ are the realization of random variables $G_1,\ldots,G_n$, respectively.

Thus, we have

$$E(Z) = \sum_{k=1}^{n} E(G_k) \text{ and}$$

$$Var(Z) = Var(G_1 + G_2 + \ldots + G_n)$$

$$= \sum_{k=1}^{n} Var(G_k) + 2\sum_{1\le k<l\le n} Cov(G_k, G_l)$$

if G_0, $G_1,\ldots,G_T$ are iid random variables, then $Cov(G_k, G_l)=0$ and $Var(G_k)=Var(G_0)$, therefore, $Var(Z)=n\ Var(G_k)=n\ Var(G_0)$, this confirms statement (1).

if $G_1, G_2,\ldots,G_T$ are correlated, since $Var(G_k) = Var(G_0)\forall k$

$$Var\ (Z) = n \cdot Var(G_0) + 2 \sum_{1 \le k < l \le n} r \sqrt{Var(G_k) \cdot Var(G_l)}$$

$= n \cdot Var(G_0) + (n^2 - n) \cdot r \cdot Var(G_0) = n(1 + nr - r)Var(G_0)$, this confirms statement (2). ð

Corollary 4.1. *In a supply chain with general product structure where the production of each component j is used to satisfy the demand for n items $i_1, \ldots, i_n$, suppose item demands are correlated random variables with coiefficient r and the same variance, the item-component demand amplification DA^s_{ij} between items $i_1, \ldots, i_n$ and component j change by a factor of* $\left(\sqrt{\dfrac{1 + nr - r}{n}} - 1 \right)$.

Proof. The item-component demand amplification

$DA^s_{ij} = CV(Z) - CV(G)$

$$= \frac{\sqrt{Var(Z)}}{\sum_{k=1}^{n} E(G_k)} - \frac{\sqrt{Var(G)}}{\sum_{k=1}^{n} E(G_k) \Big/ n}$$

From *Proposition 5*, we have $Var(Z) = n(1 + nr - r)Var(G)$, therefore

$$DA^s_{ij} = \frac{\sqrt{n(1 + nr - r)Var(G)} \Big/ n}{\sum_{k=1}^{n} E(G_k) \Big/ n} - \frac{\sqrt{Var(G)}}{\sum_{k=1}^{n} E(G_k) \Big/ n}$$

$$= \left(\sqrt{\frac{(1 + nr - r)}{n}} - 1 \right) CV(G)$$

ð

5. The Impact of Capacity Levels on Demand Amplification

We now consider the effects of binding capacity constraints. Considered in isolation from other influences, when resource capacity is binding in any time period the size of production batches tend to reduce which potentially reduces the demand variation in lower tiers of the supply chain. To pursue this intuition with more rigor, we consider a simplified case where all resource capacities are used to produce a particular item, the capacity is preset at a constant level for all periods without dynamic changes. Consider three possible capacity settings: (1) *maximum capacity,* set to the maxinum period demand in the planning horizon, denote this capacity level Cap_{max} ,(2) *averaged capacity (*or *minimum capacity),* set to total demand in the planning horizon divided by the number of periods, denote this capacity level Cap_{avg}. Note that this is the *minimum* “workable" capacity level, as any point less than this will be insufficient to maintain production in excess of demand. (3) *actual capacity,* set in between Cap_{avg} and Cap_{max}, denote $Cap_{act.}$ Clearly, Cap_{max} ³ Cap_{act} ³ Cap_{avg}. Figure 2 illustrates a representative demand pattern, along with the three capacity levels.

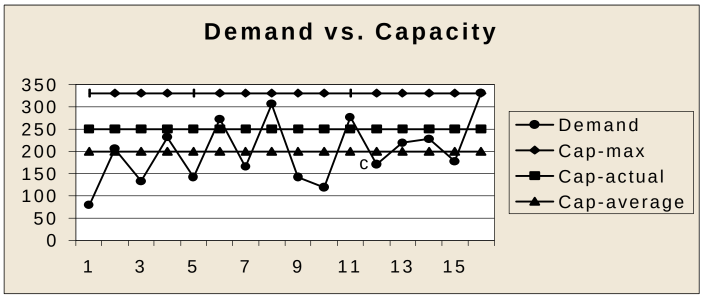

Figure 2. An Example of the Three Capacity Settings Relative to Demand

We now define a *capacity smoothing coieficient* ***e*** using the terms defined above:

$$e = \frac{(Cap_{\max} - Cap_{act})}{(Cap_{\max} - Cap_{avg})}$$

When ***e*** *= 1*, the *actual capacity* is set equal to the average capacity (which is the minimum capacity). This implies that the amount of production must be identical across all periods, and in each period the production consumes all resource capacity. This results in a complete *smoothing* of production over all periods. Note that this 100% utilization of the capacity may not be desirable since discrete lot sizes would have to be broken apart. Conversely, when ***e*** *= 0*, the *actual capacity* is set to the maximum capacity. This represents the cases where the capacity is not binding, as is the assumption for the analysis in Sections 4.2. Under this condition, there is no *smoothing* necessary due to capacity limitations. When *0*< ***e*** < 1, the actual capacity is placed between the maximum and the minimum. Furthermore, the closer the actual capacity is set to the minimum, the closer ***e*** is to 1, and the greater the *smoothing* effect is likely to be. We now present the main capacity smoothing results.

Proposition 6. *Suppose component j of item i is manufactured in a facility, which has a capacity smoothing coefficient* ***e*** *. If the setup/inventory cost ratio is not a factor for order batching (i.e., when the ratio is close to zero),* ***e*** *has the following effect on item-component demand amplification (DA^s_{ij}):*

(1) when ***e*** *=0, there is no demand amplification*

(2) when ***e*** *=1, the demand amplication is -CV(D)*

*(3) when 0<**e**<1, the demand amplication reduces by* $\left(\sqrt{c}-1\right)$, where $c = \dfrac{Var(X)}{Var(D)} < 1$

Proof. When ***e*** *=0* capacity is non-binding, thus, from *Proposition 1* there is no demand amplification ($DA^s_{ij}=0$). This confirms statement (1).

Use the notation similar to before and denote random variables $X_1,\ldots,X_T$ the production quantity for component *j* after scheduling, and random variables $D_1,\ldots,D_T$ denote the demands for item *i*. Denote $x_{j1}, x_{j2},\ldots,x_{jT}$, the occurrences of random variables $X_1,\ldots,X_T$, and $r_{i1}, r_{i2},\ldots,r_{iT}$ that of $D_1,\ldots,D_T$. For simplicity, assume $a_{ji}=1$.

When ***e*** *=1*, the production quantity for each period must be set equal to the capacity Cap_{act} which is set to Cap_{avg} , i.e., $x_{j1}=x_{j2}=\ldots=x_{jT} = \dfrac{1}{T}\sum_{t=1}^{T} r_{it} = Cap_{avg} = Cap_{act}$

Thus, E(X) = Cap_{act} =E(D) and Var(X) = 0 , therefore

$$DA^s_{ij} = CV(X) - CV(D)$$
$$= 0 - \frac{\sqrt{Var(D)}}{E(D)} = -CV(D)$$

this confirms statement (2).

When $0<$***e*** <1, the capacity is binding for at least one demand period, i.e. **$** r_{ik}, $(r_{ik}-Cap_{act}) = x_{ik} > 0$. As a result, the demand r_{ik} will be produced in two parts, $(r_{ik}-x_{ik})$ and x_{ik}. Or, more precisely, demand r_{ik} will be satisfied by at least two productions x_{jl}£$(r_{ik}-x_{ik})$ and x_{jm}³x_{ik} and $x_{jl}+x_{jm}=r_{ik}$. Essentially, to satisfy demand $r_{i1}, r_{i2},\ldots,r_{iT}$ with production $x_{j1}, x_{j2},\ldots,x_{jT}$, we need to deal with two subsets of demands, say *K* and *L*: for *k* Î *K*, we have $(r_{ik}-Cap_{act}) = x_{ik} > 0$ and for *l* Î *L*, we have $(r_{il}-Cap_{act}) = x_{il} \le 0$. In other words, we use excess capacity ($\sum_{l \in L} x_{il}$) in some periods to cover excess demands ($\sum_{k \in K} x_{ik}$) in some others. It is then easy to see that the variance for production will be smaller than the vairance for demands, i.e.,Var(X) <Var(D) if set K is nonempty. Set $c = \frac{Var(X)}{Var(D)} < 1$, then we have

$$DA_{ij}^{s} = CV(X) - CV(D)$$
$$= \frac{\sqrt{c \cdot Var(D)}}{E(D)} - \frac{\sqrt{Var(D)}}{E(D)} = \left(\sqrt{c} - 1\right) \cdot CV(D)$$

This confirms statement (3). ð

The above results suggests that when resource capacity is binding (0<ε≤1), the demands emanating from a facility will have less variation than the demand entering the facility. Considering demand amplification in a multi-item environment is more complex. Binding capacity constraints still limit the lot-sizes, but the capacity levels relative to the average demand for each item are not so tight because the capacity is established relative to the total demand for all items at that facility.

Corollary 5.1. *When overtime or outsourcing are used to increase capacity the effect of capacity smoothing reduces, i.e., the capacity smoothing coieficient* ***e*** *reduces, and the item-component demand amplificaiton increases.*

It is not difficult to see the above result since overtime or outsourcing increase Cap_{act} to be closer to Cap_{max} which in turn reduces ***e***. The intuition behind this is simply that when more capacity options are available at the upstream of the supply chain, while the overall production performance may improve, a higher level of demand amplification may result.

6. Demand Amplification for Multiple Items

Up to this point, we have been concentrating on our analysis on *non-zero production periods* (i.e., $t=1$, $v+1$,..., T-$v+1$) for the item-component pair under consideration. However, in the more general multiple-item cases, each production period may be populated with non-zero production of different components. It is of practical importance to know the *collective demand amplification* of all items in a particular supply tier. To do this, we first need to know for each item the demand variation of its production over *all* (zero and non-zero production) periods. Suppose the batch size of all items is v. Now consider a particular item-component pair (i ,j). Denote $x_{j1}, x_{j2},..., x_{jT}$ the demand stream for component j drawn from random variables $Y_1,\ldots,Y_T$, and $r_{i1}, r_{i2},\ldots,r_{jT}$ the demand stream for item i drawn from random variables $D_1,\ldots,D_T$, respectively. For any block of v periods in the planning horizon 1,...,T, there is exactly one *non-zero production period* for component j, i.e., $Y_t = a_{ji} (D_s +\ldots+ D_{s+v})$, $t<s$, and all other periods in the block do not produce j, or $Y_t=0$, we may define random variable Y as follows:

$$Y = \begin{cases} a_{ji}(D_1 + \ldots + D_v) & \text{with probability } 1/v \\ 0 & \text{with probability } (v-1)/v \end{cases}$$

Suppose $D_1,\ldots,D_T$ have expected value $E(D)$ and variance $Var(D)$, then random variables Y has the expected value $E(Y)=a_{ji}E(D)$.

Lemma 6.1. *Under the above setting, suppose the batch size is v and each production period within a v-period block has equal probability* $1/v$ *to be chosen for producing one batch of j, then*
(1) the variance for random variable Y, is $(v-1)E(Y)^2$
(2) the coefficient of variation $CV(Y)=\sqrt{(v-1)}$
Proof.

Without loss of generality, let $a_{ji}=1$, denote $m=E(Y)=E(D)$

$$Y = \begin{cases} D_1 + \ldots + D_v = v \cdot m & \text{with probability } 1/v \\ 0 & \text{with probability } (v-1)/v \end{cases}$$

$$Var(Y) = \frac{(v-1)}{v}(0-m)^2 + \frac{1}{v}(v \cdot m - m)^2 = \frac{(v-1)m^2}{v}(1+v-1) = (v-1)m^2$$

$$CV(Y) = \frac{\sqrt{(v-1)m^2}}{m} = \sqrt{(v-1)} \qquad ð$$

From *Lemma 4.1* we can conclude the following for a single item.

Proposition 7.1 *If the demands $r_{i1}, r_{i2},\ldots,r_{jT}$ are drawn from iid random variables, then the tier-to-tier demand amplification is an increasing function of the lot size as follows:* $\frac{\sqrt{(v-1)}(v-1)}{v}$.

Proof. Since the demands are drawn from iid random variables $D_1,\ldots,D_T,$ we know that $Var(Y)=a^2_{ji}\, v\, Var(D)$. Without loose of generality, let $a_{ji}=1$, denote $m=E(Y)=E(D)$ then from *Lemma 4.1* we have $Var(Y)=(v-1)m^2 = v.Var(D)$

Thus, $Var(D)=\frac{(v-1)m^2}{v}$, and $CV(D)=\sqrt{\frac{v-1}{v}}$

Therefore, the tier-to-tier demand amplification is

$$\begin{aligned} TA_{kl} &= CV(Y)-CV(D) \\ &= \sqrt{(v-1)}-\sqrt{\frac{v-1}{v}} \\ &= \frac{\sqrt{(v-1)}(v-1)}{v} \end{aligned}$$

Obviously the above term is an increasing function of the lot size $v>0$. ð

Now consider N item-component pairs, denote $D1,\ldots,DN$ random variables characterizing the item demands, and $Y1,\ldots,YN$ characterizing the corresponding component production over the planning periods. Thus the production orders in the next tier for all components is the sum of random variables $Y1+\ldots+YN$. Suppose for each component $m=E(Yi)=E(Di)$, thus $E(Y1+\ldots+YN)=Nm$. We now present the following result:

Proposition 7.2 *Under the above setting, if the item demand for all N items are iid random variables, then the following are true at the component tier*

(1) the variance of production orders is $N(v-1)m^2$

(2) the coefficient of variation is $\sqrt{\frac{v-1}{N}}$

(3) the tier-to-tier demand amplification is an increasing function of the lot size while an decreasing function of N as follows: $\sqrt{\frac{v-1}{N}}\left(1-\frac{1}{\sqrt{v}}\right)$

Proof.

Since *Y1,...,YN* are iid random variables, by applying *Lemma 4.1* the variance of production orders *(Y1+...+YN)* is

Var (Y1+...+YN)=Var (Y1)+...+Var(YN)

$= N(v-1)m^2$ this confirms statement (1).

$CV(Y1+...+YN) = \frac{\sqrt{N(v-1)m^2}}{Nm} = \sqrt{\frac{v-1}{N}}$ this confirms statement (2).

$$Var(D1+...+DN) = N \cdot \frac{(v-1)m^2}{v}$$

$$CV(D1+...+DN) = \sqrt{\frac{v-1}{vN}}$$

$$TA_{kl} = CV(Y1+...+YN) - CV(D1+...+DN)$$
$$= \sqrt{\frac{v-1}{N}} - \sqrt{\frac{v-1}{vN}}$$
$$= \sqrt{\frac{v-1}{N}}\left(1-\frac{1}{\sqrt{v}}\right)$$

The above term is an increasing function of the lot size *v*, while a decreasing function of *N*, the number of components under production. ð

6. Insights from the Analysis

To summarize the above analysis and put the main results into the right managerial context, we will make a few observations as follows.

1. ***The effects of order batching.*** When there is no incentive to consolidate batches in the supply chain there will be no item-component demand amplification. When demands are consolidated into production batches over time, the mean and variance

for a lower tier component will both increase. When demands are positively correlated, the variance increases further, while the converse is true for negative correlation. An interesting result from *Propositions 1, 2* and *Corollary 3.1* is that while the variance of a lower tier component may increase significantly after order batching, but since the mean also increases by the same factor the coefficient of variation decreases. As a result the *item-component demand amplification* reduces. While the *item-component demand amplification (Definitions 1 and 2)* measures the change in variation for a particular item-component pair, the *tier-to-tier demand amplification (Definition 3)* measures the change in variation from one supply tier to another over all items. As shown in *Propositions 7.1* and *7.2*, the *tier-to-tier demand amplification* increases from one supply tier to another as a function of the lot size. When an increasing number of items are processed simultaneously, this amplification effect reduces. However, the setup capability of the system and the degree of process similarity among the items limit the number of items that could be realistically processed at the same time.

2. ***The effects of multiple schedule releases.*** When upper tier customers make multiple schedule releases, it is preferable to follow the expected demand over all up-to-date releases rather than following any particular single release. Multiple schedule releases is a common source of frustration in manufacturing supply chains where a release may change several times by an upper tier manufacturer before actual execution. As stated in *Proposition 4*, it is advisable to follow the expected value of the multiple releases rather than following any particular one. On the other hand, as stated in *Proposition 3,* if the releases and the period-to-period demands are iid, multiple releases do not differ from single release in terms of demand amplification. These results imply that it is possible to reduce the variance created by multiple schedule releases. The key is to focus on the underlying population of schedules that will be released rather than any particular single release. Such population can be estimated using some empirical distribution (e.g., such as the simple policy stated in *Proposition 4),* or perhaps making use of historic order data to construct *a priori* distributions.

3. ***The effects of product design and component sharing.*** When the components manufactured in the supply chain are shared by a large number of upstream products, the fluctuation in end-item demands tends to create a more significant level of variance. When the product demands are positively correlated, this variance will be further exaturated. Conversely, when the demands are negatively correlated, the variance decreases by comparison. An analysis comparing the general and assembly product structures reveals the relationship between the nature of component sharing and demand variance. As stated in *Proposition 5* and *Corollary 4.1*, the more products sharing a common component the higher volume and demand variance the component experiences. Again, when putting into the perspective of total volume, the amplification effect for products with general structure is actually less significant then that of the assembly products. In the case where product demand are negatively correlated, demand fluctuations of different products tend to "cancel out" with each other, a smaller variance *and* demand amplification are to be expected for the general prodcut structure. The result here provides some insights when considering a redesign or condolidation of existing product designs in the supply chain to include common components. For instance, if the end-product demands tend to be positively correlated (such is the case in the computer industry) the demand variance will amplify through the supply tiers and prompting an unreasonably high level of reserved capacity in downstream facilities.
4. ***The smoothing effects of limited capacity.*** When the effects of order batching is excluded from consideration, limited capacity has a smoothing effect on demand amplification. When manufactured in a capacity-bounded facility, the orders eminated from the facility will invariably have a lower variance than the orders entering it. This phenomenon explains a regularly (artificially) smooth order pattern commonly observed downstream in the manufacturing supply chain despite of the fluctuation in end-item demand. This suggests that the supply chain behaves in a more stable and consistent manner over time when its capacity is close to its true expected demand. Setting capacity based on peak demands, for instance, is likely to create a much higher level of variance in the system. However, the capacity issues

become much more complex when strict product due-dates are imposed and various expediting and outsourcing activities come into play.

7. Conclusions

Demand propagation is a fundamental behavior of manufacturing supply chains, a deeper understanding of this phenomenon is essential to the overall improvement of its performance. In this paper, we present the idea that using the basic logic of which the production information system use to drive supply chain operations, some important aspects of supply chain behavior can be explained by fundamental principles. The study focuses on a make-to-order environment where the production scheduling system is used to link production plans across the tiers in a supply chain, a typical mechanism of inter-facility, inter-supplier production management in automotive and electronic industries today. In a related study (Meixell and Wu, 1998) the main analytical results are tested empirically using Monte Carlo experiments.

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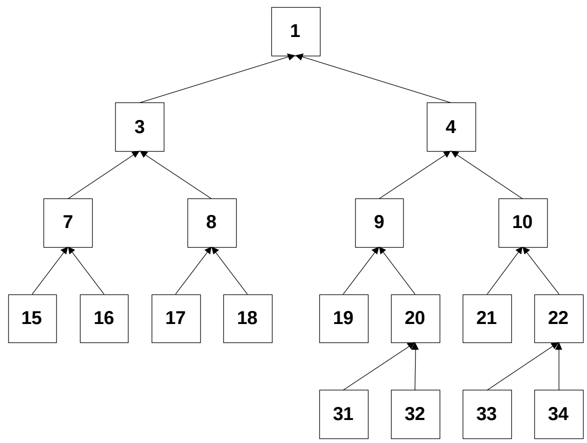
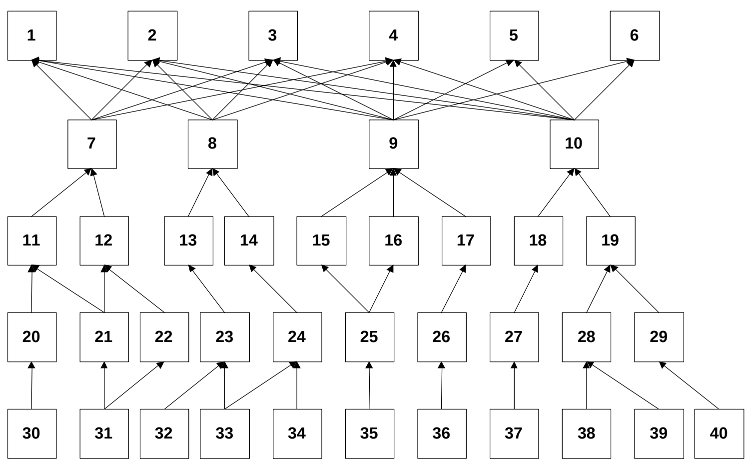


(Assembly Product Structure)



(General Product Structure)

Figure 1. Examples of an Assembly and a General Product Structure (Tempelmeier and Derstroff, 1996)