Merit Aid Management and Competition in the University Marketplace*

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Abstract

U.S. colleges and universities are increasingly turning to merit aid offers as a competitive tool to attract better students, with the level of merit aid offers increasing from $1.2 billion in 1994 to $7.3 billion in 2004. Although the total amount of merit aid offered has increased, universities vary dramatically in the amount of merit aid they use to attract these top candidates. While it would seem intuitive that the better (and wealthier) universities would offer more merit aid, the top-ranked universities actually offer far less merit aid than do others, and some top schools offer no merit aid at all. In this paper, we construct a theoretical model to explain this phenomenon. We demonstrate that it is not the quality of the university *per se* that drives the negative relationship between university quality and merit aid offers. Rather, two other factors drive this negative relationship: (i) the differences between the quality levels of closely-competing universities; and (ii) the universities' values of its applicants. We show empirically that the differences in quality levels of the top universities are greater than the differences in quality levels of lower-ranked schools, i.e., differences in quality levels among schools is positively related with quality itself. With this empirical relationship in place, we show that the greater the differences in quality levels of universities engaged in a particular quality-local competition between schools with similar ranks, the smaller the average merit aid offer among these local competitors. We also show empirically that a university's value of a particular candidate less its value of a "safety" candidate is decreasing in university quality. We use that result in our model to show that better universities also offer less merit aid in part because they have better safety candidates. We discuss the theoretical and managerial implications of our results and their implications for price management in markets beyond the university domain.

Keywords: Merit aid management, university competition, Nash equilibrium, revenue management, game theory, pricing

Squeezed on one side by state universities, whose tuition is a tiny fraction of what private colleges charge, and on the other by elite private institutions like Yale, Princeton or Amherst, private liberal arts colleges like Allegheny are routinely offering merit aid to students these days. Such scholarships are particularly pervasive in the Midwest, where many liberal arts colleges award them to as many as half or even three-quarters of their students…. The result is a college pricing system that can feel as varied, or even mysterious, as buying airplane seats, with students sometimes shopping for the best deal. University officials, defending the era of $30,000-a-year tuitions, speak of a "sticker price" and "discount price" and note that many students do not pay close to the full costs of tuition…. So prevalent has the practice become that over the last decade, the amount of money granted in merit scholarships nationally grew to $7.3 billion in 2004 from $1.2 billion in 1994, said Kenneth E. Redd, director of research and policy analysis at the National Association of Student Financial Aid Administrators.

New York Times, Jan 1, 2006, p. 1, "Aid Lets Smaller Colleges Ask, Why Pay for Ivy League Retail?," Alan Finder

Financial aid once went to the poorest kids. Now, grants awarded for academic merit or special talent in sports or the arts are growing faster than grants based on need. States spend 25% of their scholarship money on merit awards, up from 10% a decade ago, while private colleges have gone to a 36% merit share, up from 27%. Private colleges have always used merit aid to round out their orchestras or sports teams, of course. But now they increasingly see merit aid as a way to help them win the ratings-guide race and to "shape" a freshman class by, for example, recruiting science majors. Fifteen states, meanwhile, are using merit scholarships to lure bright in-state students to their local universities. The states calculate that the tactic will motivate high schoolers and raise the rates of those going to college, keep educated young people in-state after graduation, and make themselves more attractive to employers. Florida and Georgia are finding their merit-aid programs hugely expensive, but politically difficult to scale back. Even so, another half-dozen states are looking at their own merit plans.

Wall Street Journal, January 31, 2005, p. R4, "More Students, Higher Prices, Tougher Competition," June Kronholz

At the meeting [of the presidents of Amherst, Williams, Swarthmore, Barnard and seven other selective liberal arts colleges] in New York, the presidents said they spelled out their concerns over families' paying of thousands of dollars for private college counselors, obstacles for low-income applicants and tactics some colleges use to rise in the U.S. News & World Report rankings. They spoke of efforts to drive up a college's number of applications, so it can turn away a greater proportion of students and appear more selective, or to distribute merit aid to lure students who are top notch but not financially needy.

New York Times, September 19, 2006, p. 16, "Princeton Stops Its Early Admissions, Joining Movement to Make Process Fairer," Alan Finder

1. Introduction

The quotes above reflect a vibrant marketplace for higher education in which universities compete with one-another for students, faculty, prestige, and financial resources. The widely cited university rankings, such as the general undergraduate rankings by *U.S. News and World Report's America's Best Colleges* and the *Wall Street Journal*, provide highly visible scorecards summarizing the results of that competition (roughly analogous to stock prices or market capitalization for publicly traded corporations). The salience of these rankings makes it imperative for universities to adopt strategies to improve their ranks. One of the most striking facts about the university race to improve ranks is the increasing emphasis on the tactical use of merit aid: the amount of money granted in merit scholarships nationally grew to $7.3 billion in 2004 from $1.2 billion in 1994.

As the university marketplace becomes more competitive, merit aid is becoming a potent tool that universities are using to price discriminate and attract better students. (Differing merit aid offers is the practice of third-degree, or multi-market price discrimination; see Tirole 1988.) Kane (1999, p. 80-81), makes an interesting point about merit aid and price discrimination:

> *In many industries, differing prices for different buyers of the same product are taken as a sign of market power. However, in higher education, such price discrimination is a result of the declining market power of colleges. Competition tends to force an institution's prices closer to its costs. But because each student adds a different amount to the value of his or her classmates' degrees, the net cost of educating each student is different even if the cost of the bricks and mortar is the same.*

With the increased popularity of the rankings publications, and the role that student quality plays in those rankings, the substantial jump in merit aid offers is easy to explain. For example, while Fallows (2003) is critical of the rankings publications, he admits that rankings have promoted an educational meritocracy in which the best students, versus lower quality legacy students (whose relatives attended the university previously), are more likely to be accepted by the top universities. Thus, universities should offer financial enticements to attract the best students (see Rothschild and White 1995); and, hence it seems obvious that the best students (who tend to apply to the best universities) should receive the most merit aid (as depicted by the downward sloping line in Figure 1). However, the best students do not necessarily receive the most merit aid. In the university marketplace, the best students tend to be matched with the best universities, and top-ranked universities actually offer less merit aid than do other highly-ranked, albeit not top-ranked, universities (Geiger 2004). (See Figure 1 again, focusing on the actual link between university rank and merit aid and the supporting data in Appendix Table A1.) In fact, some top-ranked universities (including all eight Ivy League schools) offer very little or no merit aid. Hence, those top student candidates who choose to attend Ivy League universities, do so largely without the benefit of merit aid. Rather than seeing the better universities offer more merit aid to attract these top students than their lower-ranked peers, they actually offer less.

[Insert Figure 1 about here]

This empirical phenomenon is puzzling: the competition among the top-10 universities, for example, to attract very-high-achieving high school seniors should be the same or more intense than the competition among other highly-selective schools to attract high-achieving high school seniors. But, it is not. In this paper, we develop a model to explain why merit aid offers tend to be decreasing in university quality, and thus provide insights into this puzzle.

The phenomenon seems to be consistent with recommendations in the pricing literature in marketing (e.g., Nagle and Holden 2002), where low quality brands and products have lower prices (i.e., in our case, offer higher merit aid). Empirically, the dispersion in tuition among universities is much lower than the dispersion in merit aid offers; thus the relative price paid by the students is largely determined by their merit aid offers. However, this explanation is ultimately unsatisfactory as merit aid does not continue to increase as the rank of the university decreases (i.e., becomes more poorly ranked). In fact, merit aid tends to increase at a decreasing rate with rank and appears to level off.

To explain this curious observation, we construct a game-theoretic model in which each university's objective in the management of its merit aid is to attain its best possible rank. In calculating its optimal merit aid offers, each university determines: (i) its value of each candidate, where its values of candidates differ by SAT scores, class ranks, and the like, (ii) the competitors to which the university believes the student has applied and has been accepted; (iii) the university's beliefs about the merit aid offers of the universities that have accepted the candidate; and (iv) each candidate's preferences. We then focus on how the equilibrium merit aid offers to each applicant depend on the qualities of the universities engaged in the competition to attract the candidate, the dispersion of qualities among the competitors, and the universities' values of the applicant.

Our analysis rests on the conjecture that the university marketplace is not one large market, but rather a series of "quality-local" markets, where quality-local implies competition among university of same quality, i.e., rank. In these quality-local markets, the universities at the top compete almost exclusively with only one another to attract the top candidates, and similarly for those universities farther down in each competitive quality-local market. For example, while the Ivy League universities Harvard and Yale compete with one another, and Patriot League universities Colgate and Bucknell compete with each other, Harvard and Colgate largely do not compete for the same students. We might expect, then, that the Ivy League schools would behave toward their target students much the same way that the Patriots League Schools would behave in competing in the pool of very talented, albeit not top, candidates. (That would lead to a horizontal line in Figure 1.) In fact, since the Ivies are competing for better students and are more costly, one might expect them again to offer more merit aid. (Note that the Ivies offer generous need-based aid packages.)

For the merit aid offers to be negatively correlated with quality, something about the marketplace that includes the top universities must be fundamentally different from the marketplace that includes high-quality, albeit not top, universities. We note two differences. First we note that the dispersion of competitor quality (i.e., how close in quality the universities engaged in the competition are to each other) is positively correlated with the quality of a university, i.e. the dispersion of the quality of the top ranked universities (e.g., the Ivy League universities) is greater than that of the lower-ranked, albeit high-quality, universities (e.g., the Patriot League universities). Second, better universities reject many students that are very close in quality to those they accept: with stronger outside options, higher-quality universities have a reduced incentive to offer merit aid.

We proceed as follows. In Section 2 we set up the model. In Section 3 we develop the theoretical results that we sketched above. In Section 4, we provide some empirical validation of our findings. In Section 5, we discuss our model, results, and what the theoretical and practical implications of these findings are.

2. The Model

When universities compete for ranks, every university (except the one top school and the lowest-ranked school) competes most closely with the university or universities directly above them and the university or universities directly below them. Depending on the quality of the applicant, sometimes a particular university is the top school that a candidate is considering, sometimes the university is in the middle rank, and sometimes the university is the lowest-quality (i.e., "safety") school. As Winston (1999, pp. 80-81) states:

> *Competition among schools appears to be limited to overlapping 'bands' or segments of similarly wealthy schools within the hierarchy (with the further separation by geography and ideology). As one observer puts it, 'A school competes with the ten schools above them and the ten schools below them, even if there are more than 3,300 in the country.'*

Our model accommodates this form of high-middle-low competition by considering three schools (1, 2, and 3) with university 1 being the highest quality and university 3 being the lowest quality. With three universities, we can examine how an increase in the dispersion of the quality (i.e., an improvement in the quality of university 1 and/or a worsening of quality of university 3) affects the optimal merit aid offers of the universities.

The timing of each university's admissions and merit-aid decision process is depicted in Figure 2. We assume each university receives applications and evaluates the candidates prior to our analysis. In our model, each university determines the optimal merit aid it will offer to each candidate, should the university choose to accept him or her. Outside of our analysis, each university determines which candidates to accept. Finally, and also outside of our analysis, the candidate collects all offers and decides which university to attend.

[Insert Figure 2 about here]

The University's Decision Problem

While the decision process of the three universities about how much merit aid to offer is identical, the universities can make different merit aid decisions. Before university *i*, $i \in \{1, 2, 3\}$, makes its merit aid offers, it does the following:

- The university considers the candidates' quality attributes (e.g., high-school class ranks and SAT scores) and scores candidates (i.e., values them monetarily) according to those attributes.

- The university formulates its belief about the merit aid offers that are made by its competitors. (In our analysis, these beliefs are consistent with equilibrium offers.)
- The university models the candidates' utility functions and choice probabilities.

i. Candidate Quality Attributes and Acceptance. In our model, each university partitions its acceptable applicants into two groups: desirable candidates, for whom the university must compete to attract; and safety candidates, whom the university attracts, at full tuition, with probability one. We let *g* denote a representative desirable candidate. The set of desirable candidates that university *i* accepts is M_i, and the number of desirable candidates in set M_i is m_i. The university uses its safety candidates to fill the slots that it cannot fill with its desirable candidates. Let m_{io} denote the number of safety candidates that university *i* must accept to fill its class. Let z_i denote university *i*'s number of slots.

Let v_{ig} denote the value of candidate *g* to university *i*; and let v_{io} denote value to university *i* of one of its safety candidates. We assume $v_{ig} > v_{io}$.

ii. The Competitors' Merit Aid Offers. To simplify our analysis without qualitatively affecting our results, in our equilibrium analysis, each university believes that its competitors set their Nash equilibrium offers. (As is standard in these types of analyses, the Nash equilibrium offers are common knowledge.) We let y_{ig} denote university *i*'s merit aid offer to candidate *g*. Note that we can interpret $y_{ig} > 0$ as merit aid, and $y_{ig} < 0$ as a tuition premium (i.e., a contribution to the university required to secure admission). In our analysis, we do not examine a university's determination of its list-price tuition. Rather, we focus on the discounts (i.e., merit aid) that are offered to desirable candidates. Effectively, we assume that the three universities set the same list price tuition; column 3 in Table A1 provides rough support for this assumption.

iii. The University's Model of the Candidates' Utility Functions and Choice Probabilities. Avery and Hoxby (2006) report that among high-aptitude students (e.g., students desired by the most universities), three factors drive a candidate's attendance decision: the quality of the school, the merit aid offer, and the value of the match in the candidate's mind between the university and the candidate. Thus, we model each candidate's utility of a university as a function of three variables: x_i, which denotes the quality of the university (as determined by, for example, the university's rank and its graduate school placements); y_{ig}, the merit aid offer the university offers the candidate; and ε_i, which represents the internal (and unobserved) value of the match to the candidate of the university. We assume that all candidates have the same value, x_i, of university *i*, and it is known with certainty. The merit aid offer is a decision variable. In terms of the popular rankings, university *i*'s quality, x_i, could be viewed as a moving average of previous years' ranks. In our model, we assume that the students in the year of our analysis who choose to attend university *i* do not affect x_i. Since most students base their matriculation decisions on current and past student bodies, and not on those who choose to attend in the current round of admissions, this assumption seems to be appropriate. We assume each candidate's match value, ε_i, is private to the candidate and independent across candidates (i.e. ε_i can be viewed as capturing the uncertainty that the university sees in the value that the candidate places on that university). Hence, we assume that it is a random variable with mean 0 and constant variance.

With the calculation of x_i, it is clear that one of the purposes of the rankings is to provide an approximation of that value, since it incorporates attributes that should be important in valuing a university such as the average SAT scores of the students attending, the placement rate of the university's graduates in top graduate and professional programs, the retention rate and the like.

Hence, we model a university's estimate of each candidate's utility of the university and its competitors as:

$$u_{ig} = x_i + y_{ig} + \varepsilon_i . \qquad (1)$$

In the candidate's choice problem, the candidate has chosen to apply to, and has been accepted by, universities 1, 2 and 3. Thus, at the point in time when the candidate determines which university to attend, the consideration set has already been determined. If we assume that ε_i, has a double exponential distribution, then the candidate's choice can be specified as a multinomial logit model, consistent with the empirical work on the university candidate attendance decision by Avery *et al.* (2004) and Avery and Hoxby (2006).

By applying to the universities, the candidates have expressed their interests in going to college, so we assume all candidates attend a university. Hence, given the three universities the candidate has been accepted to and is considering, the probability candidate *g* selects to attend university *i*, $q_i = prob\{u_i \geq u_j, \forall j \neq i\}$, is

$$q_i\left(x_1 + y_{1g}, x_2 + y_{2g}, x_3 + y_{3g}\right) = \frac{\exp\left(\left(x_i + y_{ig}\right)/\mu\right)}{\sum_{j \in \{1,2,3\}} \exp\left(\left(x_j + y_{jg}\right)/\mu\right)}. \qquad (2)$$

With university 1 as the best of three universities, and university 3 the worst, we have $x_1 \geq x_2 \geq x_3$. In our analysis, we examine the dispersion of the quality of the universities. For the three-university case, we define an increase in the dispersion as an increase in both $(x_1 - x_2)$ and $(x_2 - x_3)$.

The University's Expected Score Function, Decision Problem, and Nash Equilibrium. Most university rankings in the popular press rely on multi-attribute models. For example, the *US News* rankings are based on 15 different university attributes, such as university acceptance rates, graduation rates, faculty salary, alumni giving and the like. Some combination of these attributes lead to the overall rank of the university (for details on how *US News* combines attributes, see http://www.usnews.com/usnews/edu/college/rankings/about/index.php).

In our stylized analysis, we develop a two-attribute model that retains the essential features of the popular press ranking systems. We use the two generic attributes of prestige and resources to subsume the main attributes used by popular press publications. For example, resources can be seen as representing financial resources, faculty resources, and alumni giving in the case of *US News*. We write the attribute scores for these two attributes (prestige and resources) in monetary values in order to characterize the opportunity cost of increased merit-aid spending in terms of reduced resources. In our model, as in the actual popular rankings, the university receives an overall score, which is a weighted sum of attribute scores. We assume that each university's objective in setting merit aid offers is to maximize its expected overall score.

University *i*'s prestige score depends on the candidates who attend the university and the monetary value of those candidates to the university. University *i*'s expected prestige score is the sum over those candidates that the university accepts for admission of: (i) the university's value of the candidate, times (ii) the probability the candidate attends the university,

$$\sum_{g \in M_i} v_{ig} q_i(x_i + y_{ig}, \ldots) + m_{io} v_{io} . \tag{3}$$

University *i*'s resources score is its budget, B_i, less the amount it spends on merit aid. University *i*'s expected resources score is then its budget less its expected merit aid expenditures:

$$B_i - \sum_{g \in M_i} q_i(x_i + y_{ig}, \ldots) y_{ig} .$$

In making its acceptance decision, we assume for that the university must fill its class in expectation only, and that to do so it must accept safety candidates. That is, university *i*'s acceptance decisions must satisfy

$$\sum_{g \in M_i} q_i(x_i + y_{ig}, \ldots) + m_{io} = z_i . \tag{4}$$

University *i* attaches weight w_p to its prestige score and weight w_r to its resources score. University *i*'s expected overall score is

$$E[s_i] = w_p \left(\sum_{g \in M_i} v_{ig} q_i(x_i + y_{ig}, \ldots) + m_{io} v_{io} \right) + w_r \left(B_i - \sum_{g \in M_i} q_i(x_i + y_{ig}, \ldots) y_{ig} \right) . \tag{5}$$

University *i* determines its optimal merit aid offer, y_{ig}^*, for each candidate *g*, so as to maximize its expected score, (5), subject to its class size requirement, (4). The solution to this optimization program, $y_{ig}^{*}\left(x_i, x_j + y_j, x_k + y_k\right)$ for each candidate g, is school i's best-response function (i.e., its optimal merit-aid offer as a function of the sum of each competitor's quality and merit-aid offer), i.e., optimal offer functions.

We examine the Nash equilibrium of the merit-aid offer game to attract a particular candidate, given that the candidate has been accepted by the three universities under study. In the Nash equilibrium, each university sets its optimal merit aid offer: $y_{ig}^{*}\left(x_i, x_j + y_{jg}^{*}, x_k + y_{ig}^{*}\right)$. (We also examine a case in which a candidate has been accepted by only two of the universities.)

3. Results

We begin with the characterization of university i's optimal merit aid offer, y_{ig}^{*}, for each candidate, g. We then move to Lemma 1, where we establish the conditions under which better universities are more like to attract candidates (i.e., $q_1 \geq q_2 \geq q_3$). In Theorem 1, we examine the effect of an increase in the dispersion of the quality of the universities on the equilibrium merit aid offers. The dispersion of the qualities of the universities affects the competitive landscape, and thus the strategic pricing elements. In Lemma 2, we demonstrate that the relative qualities, and not the absolute qualities, of universities affect their optimal merit aid offers. In Theorem 2, we analyze the effect of the change in a university's net value of a candidate on the equilibrium merit aid offers.

We now characterize the optimal merit aid offer that university i makes to a candidate g. Directly from the first-order conditions of maximizing (5) subject to (4), we have that school i's optimal offer satisfies,

$$y_{ig}^{*}\left(x_i, x_j + y_{jg}, x_k + y_{kg}\right) = \frac{w_p}{w_r}\left(v_{ig} - v_{io}\right) - \frac{\mu}{1 - q_i\left(x_i + y_{ig}^{*}, x_j + y_{jg}, x_k + y_{kg}\right)}. \tag{6}$$

Since each school's constrained expected score function (i.e., the Lagrange function created by (5) subject to (4)) is strictly quasi-linear, expression (6) constitutes a necessary and sufficient condition for the optimal merit aid offer.

The pricing expression (6) states that the optimal merit aid offer equals: the university's net monetary value of candidate *g* to the university, $(w_p/w_r)(v_{ig} - v_{io}) > 0$; less the strategic pricing element, $-\mu/(1-q_i) < 0$. Examining this strategic pricing element, $\mu/(1-q_i)$, we have that the optimal merit aid offer is decreasing in the probability of matriculation, q_i. Hence, *ceteris paribus*, if a university attracts a candidate with a greater probability, the university offers less merit aid. Two of these terms – the net dollar value of the candidate and the strategic pricing element – form the basis of our analysis.

In Lemma 1, we establish the conditions under which university 1 attracts the candidate with a probability that is greatest, and university 3 attracts the candidate with a probability that is smallest.

Lemma 1. *In equilibrium,* $q_1 > q_2 > q_3$ *if and only if*

$$x_1 + \frac{w_p}{w_r}(v_{1g} - v_{1o}) > x_2 + \frac{w_p}{w_r}(v_{2g} - v_{2o}) > x_3 + \frac{w_p}{w_r}(v_{3g} - v_{3o}). \tag{7}$$

Proof of Lemma 1. The proof of the lemmas and theorems are in Appendix A2.

In our development, we make two assumptions about the better universities: (1) candidates perceive them as being of higher quality (i.e., x_i is objectively greater as *i* decreases—when schools are higher rated and candidates perceive them as better); and the candidates they reject are closer in quality to the candidates they reject (i.e., $(v_{ig} - v_{io})$ is smaller as *i* decreases and the schools become better). Lemma 1 states that for university 1 to have the greatest acceptance probability and university 3 the smallest, the quality differences, as measured by the x_i's, must be greater than the differences in the net benefits of the accepted candidates, as measured by $(w_p/w_r)(v_{ig} - v_i)$. In Section 4, we offer empirical support for both these assumptions, supporting the conditions in expression (7).

We now move to Theorem 1, where we present our first key finding: the sum of merit aid offers by competitors is decreasing in the quality of the best school and increasing in the quality of worst school. Hence, with an increase in the dispersion of quality among the quality-local competitors increases (i.e., the best school becomes relatively better and the worst school becomes relatively worse), the average of the merit aid offers by the universities decreases.

Theorem 1. *Consider* $x_1 + (w_p/w_r)(v_{1g} - v_{1o}) \geq x_2 + (w_p/w_r)(v_{2g} - v_{2o}) \geq x_3 + (w_p/w_r)(v_{3g} - v_{3o})$. *In equilibrium, the average equilibrium offer to candidate g by the three universities,*

$$\left(y_{1g}^* + y_{2g}^* + y_{3g}^*\right)/3,$$

Microsoft Word.lnk *is decreasing in the quality of the highest-quality university, and increasing in the quality of the lowest-quality university.*

The driver of the Theorem 1 result is that with changes in university quality, better universities make greater adjustments to their merit aid offers than do poorer ones. That is, *ceteris paribus*, a university's optimal merit aid offer is strictly decreasing and strictly concave in its own quality (see Lemma A1 in Appendix 2). To understand the greater adjustments by higher-quality universities, examine the relationship between the quality of a school and its strategic pricing element, $(\mu/\beta)/(1-q_i)$, as expressed in (4). The strategic pricing element itself is related to the merit-aid elasticity of demand:

$$\frac{\partial q_i}{\partial y_i}\frac{y_i}{q_i} > 0, \tag{8}$$

which in the logit model is:

$$\frac{\partial q_i}{\partial y_i}\frac{y_i}{q_i} = \frac{(1-q_i)}{\mu} y_i . \tag{9}$$

Writing the strategic pricing element, $\mu/(1-q_i)$, in terms of the merit-aid elasticity, we have:

*Strategic pricing element = 1/(merit-aid elasticity * merit aid offer).*

The merit-aid elasticity, as expressed in (9), is decreasing in the quality of the school (i.e., as the quality of a university improves, the probability that a candidate attends the university is less responsive to the university's merit aid offers). Hence, *ceteris paribus*, as quality improves, the university has a greater incentive to decrease its merit aid offer. With this greater incentive, the university's optimal merit aid offer is not only decreasing in quality, it is decreasing at an increasing rate.

To illustrate this result, assume U. Chicago and Notre Dame were adjacent universities competing for a particular candidate. Suppose U. Chicago is a higher-quality school, so in the context of our model, $x_{Chicago} > x_{NotreDame}$. If $x_{Chicago}$ increases by one unit, Chicago should reduce the merit aid it offers to the candidate; and if $x_{NotreDame}$ improves by one unit, Notre Dame should reduce its merit aid offer. U. Chicago's reduction due to an increase in its own quality is greater than Notre Dame's reduction due to an increase in its own quality.

The Theorem-1 analysis rests on the assumption that competition is quality-local in the sense that top ranked universities compete essentially with one another to attract the top candidates, as do middle and lower ranked schools. Hence, rather than one large higher-education marketplace, our analysis and results depend on the assumption that higher education is partitioned into smaller markets, where markets are defined not only by geography and university specialization (e.g., business, engineering, and music), but also by quality (see Grewal, Dearden, and Lilien 2006; Winston, 1999).

In our model (and in reality) Harvard effectively does not compete with Notre Dame. (Based on the choice probabilities in Avery *et al.* (2005), a candidate accepted by Harvard and Georgetown chooses to attend Harvard with probability 0.98.) Hence, the higher quality of Harvard does not drive the result that Harvard offers no merit aid and Georgetown does. Rather, in its quality-local competition for the very top students, Harvard and its competitors, with the wide variation in the quality of these very top universities, offer very little and, possibly no, merit aid, as merit aids does not provide incremental differentiation amongst them to their accepted candidates. Georgetown, along with its quality-local competitors, all of whom are very good, albeit not very top universities and whose quality variation is small, offer significant merit aid in their close competition for students (where that merit aid is needed to differentiate their university).

The dispersion of competitor quality among universities engaged in the quality-local competitions is positively correlated empirically with the quality of the universities (as we evaluate in the next section). Hence, our Theorem-1 result is an explanation that the dispersion of competitor quality, and not the absolute quality of the university and its competitors, drives the negative relationship between university quality and merit aid offers.

In Lemma 2, we demonstrate that the relative qualities of the universities, and not the absolute qualities, drive their optimal merit aid offers.

Lemma 2. *The relative qualities, and not the absolute qualities, affect university's optimal merit aid offers. Formally, consider two sets of university quality profiles* (x_1, x_2, x_3) *and* $(\hat{x}_1, \hat{x}_2, \hat{x}_3)$. *Ceteris paribus, if* $(x_i - x_j) = (\hat{x}_i - \hat{x}_j)$ *for each j, then for university i,*

$$y_{ig}^*(x_i, x_j + y_j, x_k + y_k) = y_{ig}^*(\hat{x}_i, \hat{x}_j + y_j, \hat{x}_k + y_k). \tag{10}$$

The Lemma 2 result, combined with Theorem 1 implies that it is not the quality of the universities *per se* that drive the result that better universities offer less merit aid. Rather, in the quality-local competition among universities, it is the dispersion of the quality of the competitors, which is positively correlated with the quality of competitors, that drives the relationship between quality and merit aid offers.

For example, suppose the only difference between the Harvard-Yale-Princeton competition for an Ivy League-level candidate and the Colgate-Bucknell-Lafayette competition for a Patriot-League- level candidate is the quality-levels of the universities. Lemma 2 says that in order to generate equilibrium merit aid offers by the Ivies for their Ivy-level candidate that are lower than the equilibrium merit aid offers by the Patriots for their Patriot-level candidate, the relative qualities of the Ivies must be different from that of the Patriots. In this sense, it is not the higher quality of the Ivies that drives their lower merit aid offers.

Our second key finding, Theorem 2, is that higher-quality universities also offer less merit aid because the candidates they reject tend to be close in quality to the candidates they accept, while for lower-ranked universities, the candidates they reject tend to be further in quality from the candidates they accept. In Section 1, we offered evidence that $(v_{ig} - v_{io})$ is decreasing in quality. Top universities, with better outside options to the candidates they accept (i.e., better safety candidates), have less need to attract their top candidates and hence make smaller merit aid offers.

Theorem 2. *Each equilibrium offer (* y^*_{1g}*,* y^*_{2g}*, and* y^*_{3g} *) is increasing in university i's net monetary value of candidate g,* $(v_{ig} - \tilde{v}_{io})$.

4. Empirical Relationships

In this section, we first examine the empirical relationship between university quality and merit aid offers. Next, we consider the empirical relationship between university quality and the dispersion of the qualities of the universities engaged in quality-local competitions. Then, we provide evidence that the difference between the quality of candidates accepted by a university and the quality rejected is decreasing in the university's own quality. Finally, we summarize.

4.1. Quality and Merit Aid

We make four observations that support our contention that better-ranked universities offer less merit aid. First, Harvard, Yale, Princeton, MIT, Yale, Columbia, Cornell and Brown – all most-selective school – report that they offer no merit aid. Second, Ehrenberg and Monks (1999) demonstrate that when universities improve their ranks, they tend to offer less financial aid. Using individual applicant data from 30 highly-selective institutions for the academic years 1988/89 through 1998/99, Ehrenberg and Monks (1999) find that an increase in rank (less favorable ranking) of 10 places leads to a reduction in aid-adjusted tuition of approximately four percent. Third, summary statistics of merit aid offers indicate that rank and merit aid offers are negatively correlated. For each university, *US News* reports the average merit aid award per student who receives merit aid as well as the percentage of students who receive merit aid. Based on the reports, as reproduced in Table A1, the mean of the average merit aid award per student who receives aid for private universities ranked 1-10 is $5,327; for ranks 11-20 is $8,339; for ranks 21-30 is $5,682; for ranks 31-40 is $11,621; and for ranks 41-50 is $11,900. The mean percentage of students who receive merit aid for private universities ranked 1-10 is 2.2; for ranks 11-20 is 7.3; for ranks 21-30 is 8.0; for ranks 31-40 is 14.2; and for ranks 41-50 is 16.5. Hence, the pattern indicates that better-ranked universities tend to offer smaller merit aid packages and to a smaller percentage of their students. Fourth, Epple, Romano and Sieg (2003) in the empirical section of their paper, find that for most-selective colleges and universities (in their analysis, the most-selective universities are the ones with the greatest list-price tuition), applicant SAT scores are negatively related to merit aid offers. This empirical result is consistent with our theoretical explanation. Students with higher SAT scores tend to attend better universities; and better schools, engaged in their quality-local competition, tend to offer less merit aid. (Epple, Romano and Sieg (2003) suggest that this negative relationship may be due to variables omitted from their analysis. In their analysis, they employ a process that aggregates all top universities into effectively one university. Epple, Romano and Sieg (2003) therefore, do not examine the dispersion of quality among these top universities.)

4.2. Quality and Quality Dispersion

We offer two types of evidence to show that the dispersion in quality among quality-local competitors is positively correlated with quality itself.

First, Avery *et al.* (2004), in building their "revealed preference" ranking of colleges and universities, use survey data in a multinomial logit analysis of the applicant choice problem (i.e., faced with a list of schools that have accepted the candidate, the person must choose among the schools).

Their survey of 3,240 high-achieving from the class of 2004 includes questions about the schools that accepted applicants, the school the applicant chose to attend, and financial aid offers. Avery *et al.* write the probability that candidate g attends university i, if accepted at the set S_g universities, as

$$q_{ig} = \frac{\exp\left(\theta_i + x_{ig}{}'\delta\right)}{\sum_{j \in S_g} \exp\left(\theta_j + x_{jg}{}'\delta\right)}. \tag{11}$$

In this specification, "the θ_i s embody all characteristics that do not vary within each college: whether it is a liberal arts college, the faculty, a rural as opposed to urban location, and so on" (Avery *et al*., 2005, p. 15). The vector of characteristics, x_{ig}, vary among admittees (e.g., legacy status and merit aid). The paper reports the percentage of posterior draws in a Markov chain Monte Carlo simulation in which one school's θ is greater than another's. The striking result of their analysis is that for schools at the top, a higher-ranked school's θ is greater is greater than that of a lower-ranked school in a very large percentage of the posterior draws. For example, in the competition between Harvard and Yale, Harvard wins in 98 percent of the draws; and in the competition between Yale and Princeton, Yale wins in 90 percent of the draws. That is, Harvard's desirability is distinct from Yale's. In terms of the multinomial logit model, the perceived quality difference among these schools dominates the idiosyncratic element of the utility function in the applicant's choice problem. For lower-ranked, albeit highly-selective schools, the idiosyncratic element of the utility function plays a greater role in the applicant's choice problem. For example, in the competition between University of Chicago and Johns Hopkins, Chicago wins in 51 percent of the draws; and in the competition between Hopkins and USC, Hopkins wins in 69 percent of the draws. As Avery *et al.* write, "As a rule, the lower one goes in the revealed preference ranking, the less distinct is a college's desirability from that of its immediate neighbors in the ranking." (p. 27)

Second, the *Wall Street Journal* university ranking, reproduced in Table A2 – which ranks schools by the placement of college and university graduates in top-5 business, law and medical schools – shows that the distribution of the rank-order is heavy-tailed. At the top of the 2004 *Wall Street Journal* ranking, Harvard placed 21.49 percent a recent class in top-5 business, law or medical schools, Yale 17.96 percent, Princeton 15.78 percent, and Stanford 10.70 percent. So, by dropping only four ranks from 1^{st} to 4^{th}, the placement rate is cut in half. For the schools ranked 21 through 24, the placement rates are virtually identical – roughly 3.6 percent.

In fact, the dispersion of top-5 professional school placement rates is so great for universities at the very top and so close for top, albeit not very top, universities that the distribution of ordered top-5 professional school placement rates is distributed according to a Pareto (Power Law) distribution, a heavy-tailed distribution. In the Pareto distribution of placement rates, small placement rates are extremely common, whereas large placement rates are extremely rare. For a placement rate, ρ, the cdf of the power law distribution is

$$\text{Prob}(p \leq \rho) = F(\rho) = 1 - \left(\frac{c}{\rho}\right)^{\alpha} \tag{12}$$

For the 50 colleges and universities in the *Wall Street Journal* ranking, the estimated coefficients of the power law distribution are $\hat{c} =$ 0.0164 and $\hat{\alpha} =$ 1.2194. Table 1 has the results of the estimates of the power law distribution. Table 1 also contains the Anderson-Darling and Lilliefors tests for normality, demonstrated that we reject the hypothesis that the distribution of placement rates is normal. (Rather, the distribution is heavy tailed.) Figure 3 contains the estimated and actual distributions of ordered placement rates.

[Insert Table 1 and Figure 3 about here]

So, the quality differences at the very top are greater than the quality differences of very good, albeit not top, schools. Hence, based on the Avery *et al.* (2004) choice probabilities and the professional school placement rates, the perceived and actual quality differences may be quite large at the top.

4.3. University and Applicant Quality

For the top universities, the quality of the candidates they accept is close to the quality of the candidates they reject. As the *Wall Street Journal* reports, "Every year, [the Ivies] reject many valedictorians and students with perfect SAT scores."[1] Advice from the *Washington Post* to college applicants, "Yale University accepted 8.6 percent of its applicants this year, an Ivy League low. Selective college admissions officers admit that they reject or wait-list many students who are just as good as the ones they accept. If the school is short on engineering majors or Idaho residents or piccolo players, applicants with those characteristics will be accepted. The rest will have to go elsewhere."[2] Princeton University, recognizing the fact the university rejects high-quality candidates has decided to increase the size of its undergraduate population by 11 percent. Defending the decision to increase Princeton University's undergraduate population, university president Shirley M. Tilghman, states, "We are turning away students who we know would be absolutely stellar Princeton students, and it's just because of our lack of spaces in the class."[3]

4.4. Summary of Empirical Relationships

We have attempted to establish the following:

- the dispersion of the qualities of universities among quality-local competitors is positively correlated with quality itself;
- merit aid offers are positively correlated with rank;
- from Lemma 2, only the relative qualities, and not the absolute qualities, of quality-local competitors drives the equilibrium merit aid offers.

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[1] Daniel Golden, "Forgotten Grads, Academics Aren't Only Keys to Ivy Schools," *Wall Street Journal*, April 25, 2003, p. A1.
[2] Jay Matthews, "10 Antidotes to College-Application Anxiety," *The Washington Post*, April 25, 2006, p. A14.
[3] John Hechinger, "The Tiger Roars," *Wall Street Journal*, July 17, 2006, p. B1.

With these empirical relationships and theoretical results, it follows that it is not higher quality *per se* that drives the lower merit aid offers among top-ranked universities, but rather the dispersion among the qualities of these universities engaged in their quality-local competition.

The evidence that the difference between the quality of applicants accepted by a university and the quality of those rejected in place is decreasing in quality serves two purposes in our research. With this evidence in place we conclude that matriculation probabilities are increasing in university quality (Lemma 1) and that better universities have better outside options, both drivers of why better universities offer less merit aid.

5. Discussion

With the competition for ranking in the university marketplace becoming ever more important, we sought to understand the role that merit aid offers play in that competition. Our model and analysis developed an explanation based on quality-local competition for the striking heterogeneity in merit aid offers across universities, specifically the observation that lower-ranked universities offer less merit aid than the more highly ranked universities. We now discuss the theoretical significance of this research, highlight its practical implications, alternative explanations of the relationship between rank and merit aid offers, and provide suggestions for extensions.

Our model relates to research that focuses on the equilibrium of price setting for quality-differentiated oligopolistic firms (e.g., Anderson and de Palma 2001) and the literature on strategic complements (see Vives, 1999, 2005). More narrowly our model builds on previous theoretical analyses of university pricing (Epple, Romano and Sieg, 2003, 2006; Rothschild and White 1995). Rothschild and White, (1995) and Epple, Romano and Sieg, (2003, 2006), develop models of university competition in which students are both consumers and inputs. Rothschild and White (1995) focus on whether the setting of tuition and merit aid in a perfectly competitive higher-education market results in an efficient allocation of students among universities. Epple, Romano and Sieg (2003, 2006), construct a general equilibrium model that is similar to that of Rothschild and White (1995), adding the effect of household income on equilibrium prices.

In the Rothschild and White (1995) model, optimal tuition less merit aid equals the student's value of the university less the student's contribution to the university and their pricing equation is qualitatively similar to ours. Our models, however, take quite different approaches to strategic interaction. In the Rothschild and White (1995) general equilibrium model, each university is atomistic in the sense that a change in the quality of a university does not affect its competitors' merit aid offers. General equilibrium analysis therefore is limited in the sense that the change in the quality of a university affects only the university's own merit aid offers and misses the impact of a university's attempt to attract better students (and faculty) on both the quality of its competitors and its competitors' reactions. In the study of the effect of university quality on merit aid offers, the strategic element of the Nash approach is central to our approach and our results.

Our paper is related to the celebrated college admissions problem, where candidates express their preferences for universities and also universities for candidates (Gale and Shapley 1962). Based on these reported preferences, a mechanism then matches candidates and universities. Two recent papers characterize centralized matching mechanisms that improve upon current governments mechanisms to match students and schools. Teo *et al.* 2001, investigate the placement based on standardized test scores of primary school students by the Singapore Ministry of Education. Sönmez and Balinski 1999, examine the placement of students into universities, also according to standardized test scores, by a central Turkish placement office. Sönmez and Balinski impose a fairness condition, which requires that in any placement mechanism, candidates with better test scores are assigned to better universities. These college matching problems are examples of more general two-sided matching problems (Roth and Sotomayor 1990).

Two assumptions concerning the Nash equilibrium merit discussion: (1) given a university's belief about the merit aid offers that its competitors will set, the university chooses its merit aid offers to maximize its expected score and (2) the university's belief about the merit offers set by its competitors must be consistent with the offers they actually set (for details see Osborne, 2004, section 2.6.) Currently, there is conversation concerning whether or not a competitor's strategy, in this case merit aid offers, should be endogenous to a model (e.g., be Nash equilibrium offers). This debate centers on the second requirement of the Nash equilibrium: the consistency of beliefs (see Shugan, 2004, 2005, and the citations therein, for discussions of endogeneity and competitive response). In the highly public, highly publicized environment we study, we believe his second assumption is appropriate.

Our research offers practical insights for the management of university merit aid offers. Specifically:

- A university's quality, relative to the universities that have accepted a candidate, is a primary driver of the university's merit aid offers. As the differences among the qualities of universities engaged in a quality-local competition increases, the average of the merit aid offers by the competitors to a particular candidate decreases. This result is consistent with the empirical results that the quality dispersion of schools involved in quality-local competition is increasing in school quality, and that better schools offer less merit aid. Note that by Lemma 2, relative, and not absolute quality, affects merit aid offers. That is, Ivy League universities offer less merit aid than do those in the Patriot League not because the Ivies are better, but rather because the dispersion of quality among the Ivies is greater than the quality dispersion among the Patriots.
- If candidates rejected by the university are close in quality to those the university accepts, then the university will make lower merit aid offers. In general, a university's merit aid

offers are decreasing in the quality difference between the candidates accepted by the university and its "safety" candidates.

- Each university's optimal offer to a particular candidate, ceteris paribus, is strictly decreasing and strictly concave in the quality of the university. In this sense, in a quality-local competition, better universities make greater adjustments in their merit aid offers to changes in their qualities.

For merit aid managers, the approach we used to develop the Nash equilibrium provides the structure for a viable pricing mechanism. In other words, Universities should:

- operationalize the qualities of the university and its close competitors (i.e., set the x values); estimate candidate preference parameters (see Avery et al., 2004, and Avery and Hoxby, 2006, for logit analyses of candidate preferences);
- set monetary values for candidates (i.e., set the v values);
- place candidates into groups according to the candidates' preferences for universities, and the university's monetary values of the candidates;
- form conjectures about which universities have accepted the candidates and the offers these universities will make;
- determine optimal offers for the candidates in each of the groups.

In estimating a candidate's utilities for various universities, schools can use their ranks, contacts between the universities and applicants (e.g., campus visits and letters), legacy status as well as other exogenous variables in estimating the candidate's probability of attending the university and its competitors. If the university does not believe its competitors will make Nash equilibrium offers, it must form beliefs about the merit aid offers its competitors will make (a topic of future research).

At a broader level, the competition to attract high-quality students and improve ranking affects not only merit aid offers, but other dimensions of university competition as well. The popular press is replete with reports of actions schools have taken to improve their ranks including strategic manipulation of admissions decisions and improving campus facilities. In manipulating admissions decisions, schools have rejected top candidates to reduce acceptance rates, have overemphasized SAT scores relative to other, softer criteria, and have accepted an excessive number of applicants by the early-admissions process.[4] In the competition to improve campus facilities, schools have spent lavishly on improving dorms and fitness centers.[5] University administrators, while criticizing the existence of published rankings in large part due to the manipulations they cause, especially the numerical rankings are often relieved when a current ranking first appears and they learn their universities are highly ranked.[6]

In Theorems 1 and 2, we offered two explanations of the empirical relationship between rank and merit aid offers. We can think of two other alternative explanations – ones that we reject. First, each university that offers no merit aid could place the same value on each accepted applicant. With these identical values in place, there is no need for price discrimination. However, it seems unlikely though that Stanford, which offers merit aid, would have differing values of its applicants, while Cornell, which reports zero merit aid, would have identical values. This leads us to a second explanation. The Ivies, which for the most part offer no merit aid, could be price fixing. Whether or not the Ivies are price fixing, if we examine universities ranked 20 through 50 in USNews – ranks which do not include the Ivies – we still have the result that better-ranked universities offer less merit aid.

Our model can be extended in at several useful ways. The first is to add two types of uncertainty. Universities are uncertain about which schools have accepted a particular candidate, and also about the amount of merit aid that these universities have offered to the candidate. Such analyses would not only add to our understanding of the merit aid process, but also to the more general literature on strategic

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[4] See Daniel Golden, "Some Schools Shun Top Grads," *Wall Street Journal*, October 25, 2001, News Special Edition, p. 5, and Avery, Fairbanks and Zeckhauser, 2003.
[5] See Jeremy Rutherford, Stu Durando and Graham Watson, "For Schools, New Arenas are a Suite Proposition," *St. Louis Post-Dispatch*, December 19, 2004, p. F01.
[6] Letter to the *New York Times* by Richard R. Beeman, Dean of the College of Arts and Sciences at the University of Pennsylvania, September 17, 2002.

complements and substitutes. Our second suggestion involves enriching the model to consider two or three rounds of admissions. Many universities have two early admissions rounds before the regular admissions round. One popular early admissions rule is that if a student is accepted early, the student commits to go to that university. As a result, with this early admissions practice, after an applicant has applied early, the university does not need to compete with other universities for that candidate. Early admissions then adds several interesting twists to our analysis. First, early admissions reduce price competition. Second, risk-averse schools may lock-in more students to avoid the risk of filling its class with lower-quality students. Third, in the competition to improve ranks, as Avery *et al.* (2003) suggest, schools may lower their admissions standards during the early rounds. Universities that do so can then reject more applicants during the regular admissions round, thus reducing their acceptance rates, increasing their selectivity and improving their overall ranking. By considering multiple rounds of admissions, the analysis of merit aid offers becomes a full-blown revenue management problem (See Aviv and Pazgal 2005; Bitran and Caldentey 2003; Talluri and van Ryzin 2004).

While we have focused on the specifics of the competition in the university marketplace, the essence of our analysis and model deals with quality-local competition in a production-limited environment where customer quality varies and has inherent value. Competition amongst other so-called "exclusive" institutions such as golf, country or other special interest, limited admission institutions where potential members or customers differ in terms of attractiveness or social capital could all be usefully studied using some adaptation of our framework (Sandler and Tschirhart 1980; Woolcock and Narayan 2000).

Indeed, however, the university marketplace is complex and of sufficient strategic importance on its own to merit significant study. We hope that this work has added a bit to our understanding of that marketplace, will kindle some additional discussion and spur further work in this and related domains.

References

Anderson, Simon P., and André de Palma (2001), "Product Diversity in Asymmetric Oligopoly: Is the Quality of Consumer Goods too Low?" *Journal of Industrial Economics*, 49 (2), 113-135.

Avery, C., A. Fairbanks, and R. Zeckhauser (2003), *The Early Admissions Game: Joining the Elite*, Cambridge, MA: Harvard University Press.

_____, M. Glickman, C. Hoxby and A. Metrick (2005), "A Revealed Preference Ranking of U.S. Colleges and Universities," NBER Working Paper 10803, http://www.nber.org/papers/w10803.

_____, and C. Hoxby (2006), "Do and Should Financial Aid Packages Affect Students' College Choices?" Working Paper, Harvard University.

Aviv, Y., and A. Pazgal (2005), "A Partially Observed Markov Decision Process for Dynamic Pricing," *Management Science*, 51 (9), 1400-1416.

Balinski, M., and T. Sönmez (1999), "A Tale of Two Mechanisms: Student Placement," *Journal of Economic Theory*, 84, pp. 73-94

Ehrenberg, R., and J. Monks, 1999, "The Impact of *U.S. News & World Report* College Rankings on Admissions Outcomes and Pricing Policies at Selective Private Institutions," NBER Working Paper 7227.

Epple, D., R. Romano, and H. Sieg (2003), "Peer Effects, Financial Aid, and Selection of Students into Colleges," *Journal of Applied Econometrics*, 18, 501-525.

_____, _____, and _____ (2006), "Admission, Tuition, and financial Aid Policies in the market for Higher Education," *Econometrica*, 74 (4), 885-928.

Fallows, J. (2003), "The New College Chaos," *The Atlantic Monthly*, November, 106.

Gale, D. and L. Shapley (1962), "College Admissions and the Stability of Marriage," *American Mathematical Monthly*, 69, 9-115.

Geiger, R. L. (2004), "Market Coordination of Higher Education," in P. Teixeira, B. Jongbloed and D. Dill, *Markets in Higher Education: Rhetoric or Reality?*, Dordrecht: Kluwer.

Grewal, R., J. A. Dearden, and G. L. Lilien (2006), "The University Rankings Game: Modeling the Competition among Universities for Ranking," Working Paper, Penn State University.

Kane, T. J. (1999), *The Price of Admission*, Washington, D.C.: The Brookings Institution

Nagle, Thomas T. and Reed K. Holden, (2002, *The Strategy and Tactics of Pricing: A Guide to Profitable Decision Making*, Englewood, NJ: Prentice Hall.

Osborne, Martin J. (2004), *An Introduction to Game Theory*, Cambridge, UK: Oxford University Press.

Roth, A., and M. Sotomayor (1990), *Two-Sided Matching: A Study in Game-Theoretic Modeling and Analysis*, New York: Cambridge University Press.

Rothschild, Michael and Lawrence J. White (1995), “The Analytics of Pricing of Higher Education and Other Services in Which the Customers are Inputs,” *Journal of Political Economy*, 103 (3), 573-586.

Sandler,Todd and John T. Tschirhart (1980) "The Economic Theory of Clubs: An Evaluative Survey" *Journal of Economic Literature*, Vol. 18, No. 4 (Dec), 1481-1521

Shugan, Steven M. (2004), “Endogeneity in Marketing Decision Models,” *Marketing Science*, 23 (1), 1-3.

_____ (2005), “Comments on Competitive Responsiveness,” *Marketing Science*, 24 (1), 3-7.

Talluri, D. and G. van Ryzin, (2004), “Revenue Management Under a General Discrete Choice Model of Consumer Behavior,” *Management Science*, 50 (1), 15-33.

Teo, C.-P., J. Sethuraman, and W.-P. Tan (2001), Gale-Shapley Stable Marriage Problem Revisited: Strategic Issues and Applications, *Management Science,* 47, 1252-1267.

Tirole, Jean (1988), *The Theory of Industrial Organization*, Cambridge, MA: MIT Press

Vives, Xavier (1999), *Oligopoly Pricing*, Cambridge, MA: MIT Press.

_____ (2005), “Complementarities and Games: New Developments,” *Journal of Economic Literature*, 43 (2), 437-479.

Winston, Gordon C. (1999), “Subsidies, Hierarchy and Peers: The Awkward Economics of Higher Education.” *Journal of Economic Perspectives*, 13 (1), 13-36.

Woolcock, Michael and Deepa Narayan (2000) "Social Capital: Implications for Development Theory, Research, and Policy" *World Bank Research Observer*, 15 (August), 225 - 249

Figure 1: Average Merit Aid Offers By US News Ranking Decile

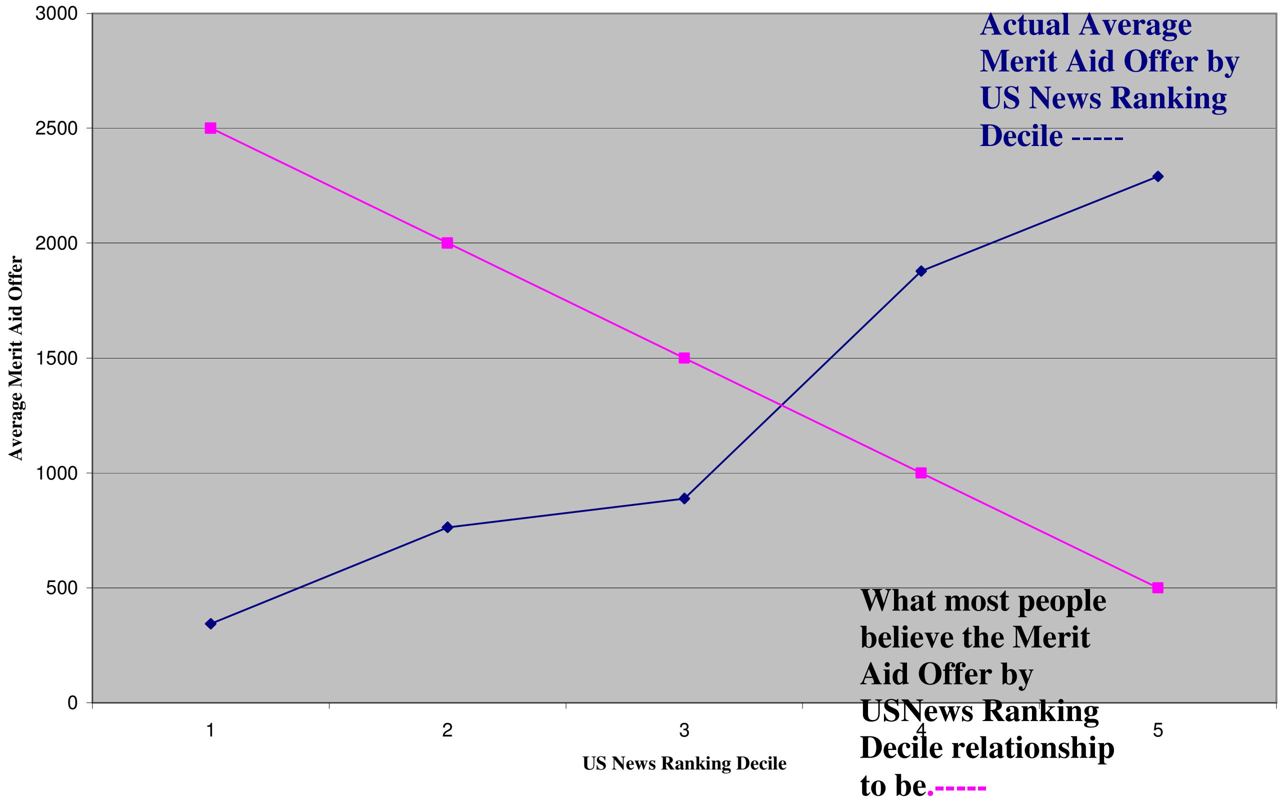
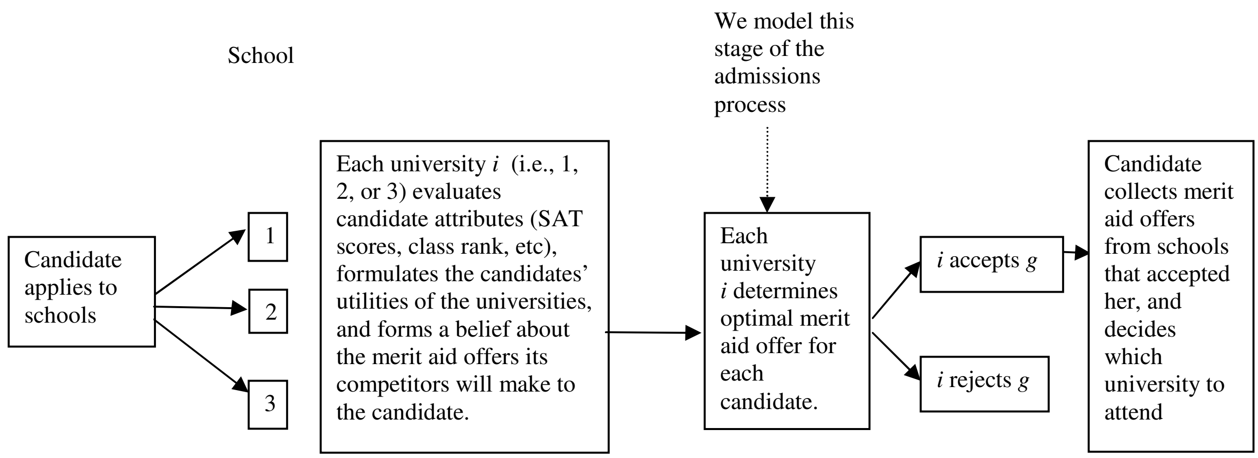


Figure 2. The sequence of events and the stages of our analysis



NOTES: In the merit aid and admission process, the candidate applies to universities 1, 2 and 3. Each university then gathers data needed to determine its optimal merit aid offer to the candidate. The university then determines the offer. Next, the university decides whether to accept the candidate and, if so, the university extends its admissions and merit-aid offer (if it chooses to make one). The candidate accepts the offer of one of the universities. We model the merit aid determination stage of the admissions process.

Figure 3. The empirical and estimated power law distribution of the placement rates in top-5 professional programs by the *Wall Street Journal* top-50 colleges and universities.

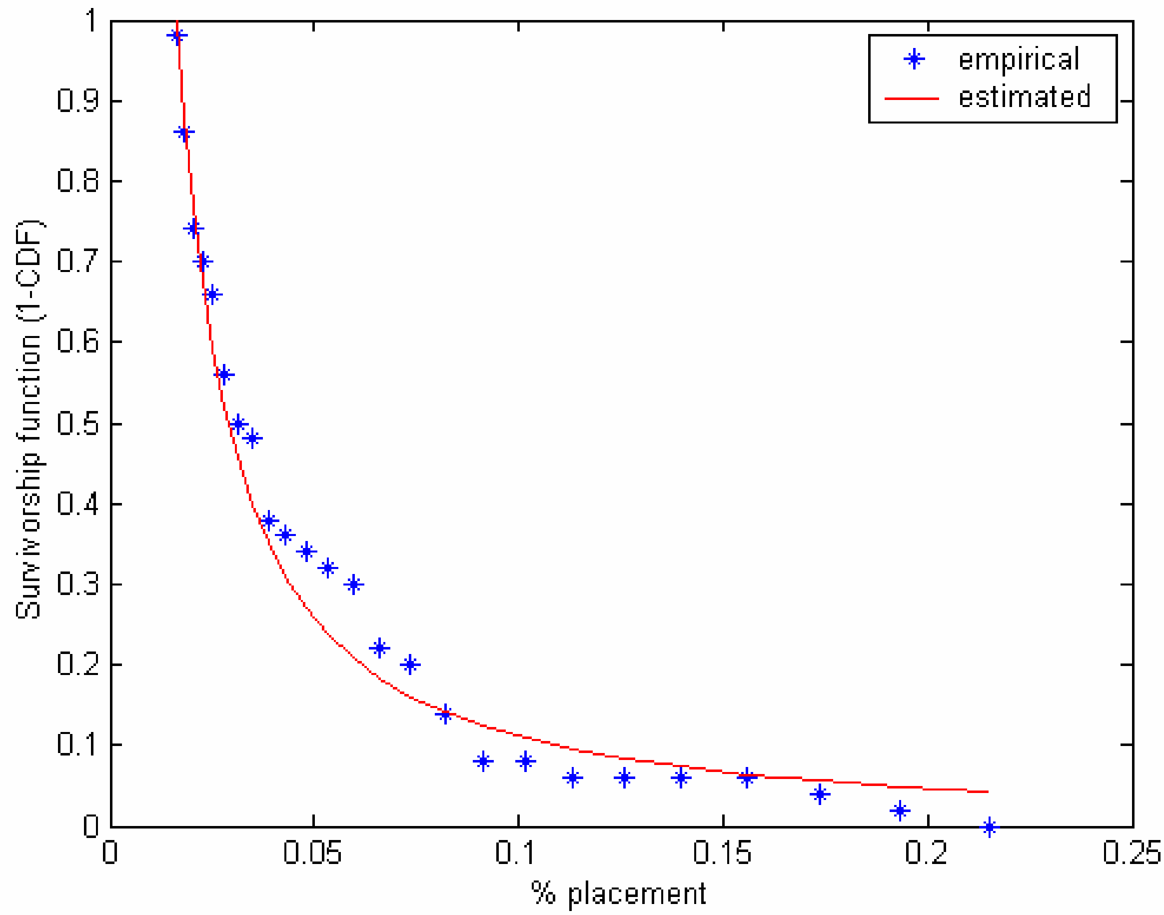


Table 1. Tests of the distribution of the placement rates by the *Wall Street Journal* top-50 colleges and universities in top-5 professional schools, showing it is-heavy tailed.

	statistic	critical value (.05)	p-value
X^2	4.7968	-	0.0909
Likelihood Ratio	13.7602*	-	0.0010
Anderson-Darling	0.6202	0.75	-
Lilliefors	0.0873	0.1266	-

* significant at p<0.05

Appendix 1: Supporting Tables – For *Management Science* Website

Table A1. 2006 USNews Rank, Tuition, Average Merit Aid Award per Student Who Receives Merit Aid, Percentage of Students Who Receive Merit Aid.

<table>
<tr><th>University</th><th>USNews Rank</th><th>Tuition 05-06</th><th>Average Merit Aid Award per Student who Received Aid: Total Undergrads</th><th>Average Merit Aid per Student who Received Aid: Decile Average</th><th>Percent Awarded Merit Aid: Total Undergrads</th><th>Percent Awarded Merit Aid: Decile Average</th><th>Average Merit Aid per Student: Total Undergrads</th><th>Average Merit Aid per Student: Decile Average</th></tr>
<tr><td>Harvard</td><td>1</td><td>32,097</td><td>0</td><td rowspan="10">5,327</td><td>0</td><td rowspan="10">2.2</td><td>0</td><td rowspan="10">343</td></tr>
<tr><td>Princeton</td><td>1</td><td>31,450</td><td>0</td><td>0</td><td>0</td></tr>
<tr><td>Yale</td><td>3</td><td>31,460</td><td>0</td><td>0</td><td>0</td></tr>
<tr><td>Pennsylvania</td><td>4</td><td>32,364</td><td>0</td><td>0</td><td>0</td></tr>
<tr><td>Duke</td><td>5</td><td>32,410</td><td>22,277</td><td>4</td><td>891</td></tr>
<tr><td>Stanford</td><td>5</td><td>31,200</td><td>3,100</td><td>10</td><td>310</td></tr>
<tr><td>Cal Tech</td><td>7</td><td>27,309</td><td>27,896</td><td>8</td><td>2232</td></tr>
<tr><td>MIT</td><td>7</td><td>32,200</td><td>0</td><td>0</td><td>0</td></tr>
<tr><td>Columbia</td><td>9</td><td>31,472</td><td>0</td><td>0</td><td>0</td></tr>
<tr><td>Dartmouth</td><td>9</td><td>31,965</td><td>405</td><td>0</td><td>0</td></tr>
<tr><td>WashingtonU</td><td>11</td><td>32,042</td><td>6,914</td><td rowspan="10">8,339</td><td>14</td><td rowspan="10">7.3</td><td>968</td><td rowspan="10">763</td></tr>
<tr><td>Northwestern</td><td>12</td><td>31,789</td><td>3,423</td><td>1</td><td>34</td></tr>
<tr><td>Cornell</td><td>13</td><td>31,467</td><td>0</td><td>0</td><td>0</td></tr>
<tr><td>JHU</td><td>14</td><td>31,620</td><td>13,016</td><td>6</td><td>781</td></tr>
<tr><td>Brown</td><td>15</td><td>32,974</td><td>0</td><td>0</td><td>0</td></tr>
<tr><td>U. Chicago</td><td>15</td><td>31,629</td><td>11,260</td><td>11</td><td>1239</td></tr>
<tr><td>Rice</td><td>17</td><td>20,160</td><td>5,335</td><td>20</td><td>1067</td></tr>
<tr><td>Notre Dame</td><td>18</td><td>31,542</td><td>7,630</td><td>2</td><td>153</td></tr>
<tr><td>Vanderbilt</td><td>18</td><td>31,700</td><td>17,758</td><td>13</td><td>2309</td></tr>
<tr><td>Emory</td><td>20</td><td>30,794</td><td>18,056</td><td>6</td><td>1083</td></tr>
<tr><td>CMU</td><td>22</td><td>32,044</td><td>11,721</td><td rowspan="5">5,682</td><td>9</td><td rowspan="5">8.0</td><td>1055</td><td rowspan="5">889</td></tr>
<tr><td>Georgetown</td><td>23</td><td>32,199</td><td>3,800</td><td>0</td><td>0</td></tr>
<tr><td>Tufts</td><td>27</td><td>32,621</td><td>500</td><td>2</td><td>10</td></tr>
<tr><td>Wake Forest</td><td>27</td><td>30,210</td><td>9,685</td><td>10</td><td>968</td></tr>
<tr><td>USC</td><td>30</td><td>32,008</td><td>12,702</td><td>19</td><td>2413</td></tr>
<tr><td>Lehigh</td><td>32</td><td>31,420</td><td>13,027</td><td rowspan="5">11,621</td><td>7</td><td rowspan="5">14.2</td><td>912</td><td rowspan="5">1878</td></tr>
<tr><td>Brandeis</td><td>34</td><td>32,500</td><td>17,454</td><td>22</td><td>3840</td></tr>
<tr><td>Case Western</td><td>37</td><td>28,678</td><td>12,650</td><td>30</td><td>3795</td></tr>
<tr><td>New York U.</td><td>37</td><td>31,690</td><td>6,924</td><td>11</td><td>762</td></tr>
<tr><td>Boston C.</td><td>40</td><td>31,438</td><td>8,051</td><td>1</td><td>81</td></tr>
<tr><td>Rensselaer</td><td>43</td><td>31,857</td><td>14,700</td><td rowspan="4">11,900</td><td>16</td><td rowspan="4">16.5</td><td>2352</td><td rowspan="4">2290</td></tr>
<tr><td>Tulane</td><td>43</td><td>32,946</td><td>17,020</td><td>31</td><td>5276</td></tr>
<tr><td>Yeshiva</td><td>46</td><td>26,100</td><td>7,741</td><td>3</td><td>232</td></tr>
<tr><td>Syracuse</td><td>50</td><td>28,285</td><td>8,140</td><td>16</td><td>1302</td></tr>
</table>

The data in Table A1 was collected at the USNews ranking website, http://www.usnews.com/usnews/edu/college/rankings/rankindex_brief.php on July 19, 2006. The data reported by USNews is based on university self-reported data to the Common Data Set Initiative (http://www.commondataset.org/).

Table A2. The *Wall Street Journal* 09/25/2003 ranking of the top-50 colleges and universities, as ranked by percentage of graduating classes placed in the top-5 business, law and medical schools.

Rank	School	Class Size	# Attending	Percentage of class attending
1	Harvard University	1,666	358	21.49%
2	Yale University	1,286	231	17.96%
3	Princeton University	1,103	174	15.78%
4	Stanford University	1,692	181	10.70%
5	Williams College	519	47	9.06%
6	Duke University	1,615	139	8.61%
7	Dartmouth College	1,101	93	8.45%
8	MIT	1,187	92	7.75%
9	Amherst College	431	33	7.66%
10	Swarthmore College	336	25	7.44%
11	Columbia University	1,652	118	7.14%
12	Brown University	1,506	98	6.51%
13	Pomona College	362	23	6.35%
14	University of Chicago	948	59	6.22%
15	Wellesley College	585	35	5.98%
16	University of Pennsylvania	2,785	153	5.49%
17	Georgetown University	1,666	85	5.10%
18	Haverford College	291	13	4.47%
19	Bowdoin College	404	16	3.96%
20	Rice University	764	29	3.80%
21	Northwestern University	1,978	73	3.69%
22	Claremont McKenna College	271	10	3.69%
23	Middlebury College	660	24	3.64%
24	Johns Hopkins University	1,272	45	3.54%
25	Cornell University	3,565	115	3.23%
26	Bryn Mawr College	310	9	2.90%
27	Wesleyan University	731	21	2.87%
28	Cal Tech	249	7	2.81%
29	Morehouse College	501	14	2.79%
30	University of Michigan	5,720	156	2.73%
31	New College of Florida	113	3	2.65%
32	Vassar College	581	15	2.58%
33	University of Virginia	3,213	82	2.55%
34	US Military Academy	966	23	2.38%
35	University of Notre Dame	1,985	45	2.27%
36	Emory University	1,509	33	2.19%
37	US Military Academy	986	21	2.13%
38	Macalester College	406	8	1.97%
39	Brandeis University	815	16	1.96%
40	Bates College	417	8	1.92%
41	U. California, Berkeley	6,198	118	1.90%
42	Barnard College	588	11	1.87%
43	Trinity College	485	9	1.86%
44	Grinnell College	337	6	1.78%
45	Tufts University	1,246	22	1.77%
46	Colby College	471	8	1.70%
47	Washington University	1,709	29	1.70%
48	Washington and Lee	413	7	1.69%
49	Case Western	729	12	1.65%
50	Reed College	304	5	1.64%

Appendix 2: Proofs

Proof of Lemma 1

By (2), $q_i > q_j$ if and only if

$$\left(x_i + y_{ig}\right) > \left(x_j + y_{jg}\right). \tag{A1}$$

By (4), $q_i > q_j$ if and only if

$$\frac{w_p}{w_r}\left(v_{ig} - v_{io}\right) - y_{ig} > \frac{w_p}{w_r}\left(v_{jg} - v_{jo}\right) - y_{jg}. \tag{A2}$$

From (A1) and (A2), $q_i > q_j$ if and only if

$$x_i + \frac{w_p}{w_r}\left(v_{ig} - v_{io}\right) > x_j + \frac{w_p}{w_r}\left(v_{jg} - v_{jo}\right). \tag{A3}$$

Q.E.D.

We use Lemmas A1 and A2 in the proof of Theorem 1. In Lemma A1 we show that if a university's quality improves, it should offer less merit aid to a particular candidate; and the magnitude of the change in the university's optimal merit aid offer due to a change in its quality is increasing in the quality of the university. That is, *ceteris paribus*, a university's optimal merit aid to a particular candidate is strictly decreasing and strictly concave in the quality of the university. In Lemma A2, we demonstrate that if a competitor's quality improves, then the university should offer more merit aid to a particular candidate. That is, *ceteris paribus*, a university's optimal merit aid to a particular candidate is increasing in the quality of the competitor.

Lemma A1 *Ceteris paribus, the change in university i's optimal offer with respect to a change in its own quality is*

$$\frac{\partial y_{ig}^{*}\left(x_i, x_j + y_j, x_k + y_k\right)}{\partial x_i} = -q_{ig}^{*}\left(x_i + y_i, x_j + y_j, x_k + y_k\right). \tag{A4}$$

Lemma A2 *Ceteris paribus, the change in university i's optimal offer with respect to a change in j's quality or merit aid offer is*

$$\frac{\partial y_{ig}^*}{\partial y_j} = \frac{\partial y_{ig}^*}{\partial x_j} = \frac{q_{ig}\left(x_i + y_i, x_j + y_j, x_k + y_k\right)}{1 - q_{ig}\left(x_i + y_i, x_j + y_j, x_k + y_k\right)} q_{jg}\left(x_i + y_i, x_j + y_j, x_k + y_k\right). \tag{A5}$$

Proof of Lemmas A1 and A2

Substituting (4) into (5), we have the expected score function:

$$E[\sigma_i] = w_p\left(\sum_{g \in M_i} v_{ig} q_i\left(x_i + y_{ig}, \ldots\right) + v_{io}\left(z_i - \sum_{g \in M_i} q_i\left(x_i + y_{ig}, \ldots\right)\right)\right) + w_r\left(B_i - \sum_{g \in M_i} q_i\left(x_i + y_{ig}, \ldots\right) y_{ig}\right). \tag{A6}$$

The optimization program is

$$\max_{y_{ig}, g \in M_i} E[\sigma_i]. \tag{A7}$$

The first-order conditions, which are necessary and sufficient for a maximum, are:

$$\frac{\partial E[\sigma_i]}{\partial y_{ig}} = 0 = \left[w_p\left(v_{ig} - v_{io}\right) - w_r y_{ig}\right]\frac{\partial q_i}{\partial y_{ig}} - w_r q_i \text{ for each } g \in M_i.$$

(A6)

For the logit model, we have

$$\frac{\partial q_i}{\partial y_{ig}} = \frac{q_i(1 - q_i)}{\mu}. \tag{A8}$$

Substituting (A9) into (A8), we have the equation, which we label f_{ig},

$$f_{ig} = 0 = \left[w_p\left(v_{ig} - v_{io}\right) - w_r y_{ig}\right]\frac{(1 - q_i)}{\mu} - w_r. \tag{A10}$$

Also for the logit model, we have:

$$\frac{\partial q_i}{\partial x_i} = \frac{q_i(1 - q_i)}{\mu}, \tag{A11}$$

and

$$\frac{\partial q_i}{\partial x_j} = \frac{\partial q_i}{\partial y_{jg}} = -\frac{q_i q_j}{\mu}. \tag{A12}$$

Taking the differential of (A8) with respect to y_{ig}, y_{jg}, y_{kg} x_i, x_j, and x_k, and using (A9), (A11) and (A12), we have:

$$\begin{aligned} df_{ig} = 0 = & \left\{ \left(w_p\left(v_{ig} - v_{io}\right) - w_r y_{ig}\right)\left(\frac{-q_i(1-q_i)}{\mu^2}\right) - w_r \frac{1-q_i}{\mu}\right\} dy_{ig} \\ & + \left(w_p\left(v_{ig} - v_{io}\right) - w_r y_{ig}\right)\left(\frac{-q_i(1-q_i)}{\mu^2}\right) dx_i \\ & + \left(w_p\left(v_{ig} - v_{io}\right) - w_r y_{ig}\right)\left(\frac{q_i q_j}{\mu^2}\right)\left(dx_j + dy_{jg}\right) \\ & + \left(w_p\left(v_{ig} - v_{io}\right) - w_r y_{ig}\right)\left(\frac{q_i q_k}{\mu^2}\right)\left(dx_k + dy_{kg}\right). \end{aligned} \tag{A13}$$

Solving (A10) for $\left(w_p\left(v_{ig} - v_{io}\right) - w_r y_{ig}\right)$, and substituting into (A11), we have

$$df_{ig} = 0 = -dy_{ig} - q_i dx_i + \frac{q_i q_j}{1-q_i}\left(dx_j + dy_{jg}\right) + \frac{q_i q_k}{1-q_i}\left(dx_k + dy_{kg}\right). \tag{A14}$$

From (A14), we have

$$\frac{\partial y_{ig}}{\partial x_i} = -q_i; \tag{A15}$$

and

$$\frac{\partial y_{ig}}{\partial x_j} = \frac{q_i}{1-q_i} q_j. \tag{A16}$$

Q.E.D.

Proof of Theorem 1

Taking the total differentials of the first-order conditions, (A10), using Lemmas 2.1 and 2.2 and Cramer's rule, we have:

$$\frac{dy_{ig}^*}{dx_i} = \frac{\begin{vmatrix} q_i & \frac{q_i q_j}{1-q_i} & \frac{q_i q_k}{1-q_i} \\ -\frac{q_i q_j}{1-q_j} & -1 & \frac{q_j q_k}{1-q_j} \\ -\frac{q_i q_k}{1-q_k} & \frac{q_j q_k}{1-q_k} & -1 \end{vmatrix}}{\begin{vmatrix} -1 & \frac{q_i q_j}{1-q_i} & \frac{q_i q_k}{1-q_i} \\ \frac{q_i q_j}{1-q_j} & -1 & \frac{q_j q_k}{1-q_j} \\ \frac{q_i q_k}{1-q_k} & \frac{q_j q_k}{1-q_k} & -1 \end{vmatrix}} \tag{A17}$$

and

$$\frac{dy_{jg}^*}{dx_i} = \frac{\begin{vmatrix} -1 & q_i & \frac{q_i q_k}{1-q_i} \\ \frac{q_i q_j}{1-q_j} & -\frac{q_i q_j}{1-q_j} & \frac{q_j q_k}{1-q_j} \\ \frac{q_i q_k}{1-q_k} & -\frac{q_j q_k}{1-q_k} & -1 \end{vmatrix}}{\begin{vmatrix} -1 & \frac{q_i q_j}{1-q_i} & \frac{q_i q_k}{1-q_i} \\ \frac{q_i q_j}{1-q_j} & -1 & \frac{q_j q_k}{1-q_j} \\ \frac{q_i q_k}{1-q_k} & \frac{q_j q_k}{1-q_k} & -1 \end{vmatrix}}. \tag{A18}$$

To evaluate the sign of $d\left(y_{ig}^* + y_{jg}^* + y_{kg}^*\right)/dx_i$, we alter (A18) by setting the response by the competitors to an increase in the quality or merit aid offer of university *i*, x_i, which we label $\partial \hat{y}_{jg}/\partial x_j$ or $\partial \hat{y}_{jg}/\partial y_{ig}$, as:

$$\frac{\partial \hat{y}_{jg}}{\partial x_j} = \frac{\partial \hat{y}_{jg}}{\partial y_{ig}} = \frac{q_j}{1-q_i} q_i. \tag{A19}$$

We then rewrite (A16) as

$$\frac{d\hat{y}^*_{jg}}{dx_i} = \frac{\begin{vmatrix} -1 & q_i & \frac{q_i q_k}{1-q_i} \\ \frac{q_i q_j}{1-q_i} & -\frac{q_i q_j}{1-q_i} & \frac{q_j q_k}{1-q_i} \\ \frac{q_i q_k}{1-q_i} & -\frac{q_j q_k}{1-q_i} & -1 \end{vmatrix}}{\begin{vmatrix} -1 & \frac{q_i q_j}{1-q_i} & \frac{q_i q_k}{1-q_i} \\ \frac{q_i q_j}{1-q_i} & -1 & \frac{q_j q_k}{1-q_i} \\ \frac{q_i q_k}{1-q_i} & \frac{q_j q_k}{1-q_i} & -1 \end{vmatrix}}. \tag{A20}$$

In the remainder of the proof, we evaluate $d\left(y^*_{ig} + \hat{y}_{jg} + \hat{y}_{kg}\right)/dx_i$ and show that $sign\left|d\left(y^*_{ig} + y^*_{jg} + y^*_{kg}\right)/dx_i\right| = sign\left|d\left(y^*_{ig} + \hat{y}_{jg} + \hat{y}_{kg}\right)/dx_i\right|$. We begin by evaluating $d\left(y^*_{ig} + \hat{y}_{jg} + \hat{y}_{kg}\right)/dx_i$. In doing so, to simplify the appearance of $d\left(y^*_{ig} + \hat{y}_{jg} + \hat{y}_{kg}\right)/dx_i$, without loss of generality, we set $q_k = \alpha q_j$ for $\alpha \in (0,1]$. Using $1 = q_i + q_j + \alpha q_j$, and solving for q_j, we have $q_j = (1-q_i)/(1+\alpha)$. Evaluating (A17) and (A20), using these two equalities – (i) $q_k = \alpha q_j$ and (ii) $q_j = (1-q_i)/(1+x)$ – we write $d\left(y^*_{ig} + \hat{y}_{jg} + \hat{y}_{kg}\right)/dx_i$ as

$$d\left(y^*_{ig} + \hat{y}_{jg} + \hat{y}_{kg}\right)/dx_i = \frac{\alpha q_i\left(1+\alpha^2+3\alpha+\alpha q_i^2+\alpha^2 q_i^2+q_i^2-8\alpha q_i-4q_i-4\alpha^2 q_i\right)}{\begin{pmatrix} 1+4\alpha+5\alpha^2+4\alpha^3+\alpha^4+2\alpha^2 q_i-5\alpha^2 q_i^2 \\ \\ -q_i^2-2\alpha q_i^2+2\alpha^2 q_i^3-2\alpha^3 q_i^2-\alpha^4 q_i^2 \end{pmatrix}}. \tag{A21}$$

To evaluate (A21), we evaluate the denominator, *den*, and the numerator, *num*, separately.

We now show that *den* > 0 for any $\alpha \in (0,1]$ and $q_i \in (0,1)$. To do so, we show that *den* is strictly concave with respect to q_i for $q_i \in (0,1)$, and that *den* > 0 if either $q_i = 0$ or $q_i = 1$. To demonstrate concavity,

$$\frac{d^2 den}{dq_i^{\,2}} = -\left[2+4\alpha+10\alpha^2+4\alpha^3+2\alpha^4-12\alpha^2 q_i\right]<0 \text{ for any } \alpha\in(0,1] \text{ and } q_i\in(0,1). \qquad \text{(A22)}$$

Next, if $q_1 = 0$, then $den = 1+4\alpha+5\alpha^2+4\alpha^3+\alpha^4>0$; and if $q_i = 1$, then $den = 2\alpha+4\alpha^2+2\alpha^3>0$.

We now have that *den* > 0.

Since the denominator of (A21) is positive, the sign of (A21) depends on the sign of *num*. Evaluating *num*, we show that it is strictly concave with respect to q_i for $q_i\in(0,1)$, and has three roots. To demonstrate concavity,

$$\frac{d^2 num}{dq_i^{\,2}} = \alpha\left[-\left(8\alpha^2+16\alpha+8\right)+6q_i\left(\alpha^2+\alpha+1\right)\right]<0 \text{ for any } \alpha\in(0,1] \text{ and } q_i\in(0,1). \qquad \text{(A23)}$$

The three roots of third-order polynomial *num* with respect to q_i are:

(i) 0;

(ii) $\tilde{q}_i \equiv \left(2(1+\alpha)^2-\left((3\alpha^2+3\alpha+1)(\alpha^2+3\alpha+3)\right)^{1/2}\right)/\left(\alpha^2+\alpha+1\right)$; and (A24)

(iii) $\tilde{\tilde{q}}_i \equiv \left(2(1+\alpha)^2+\left((3\alpha^2+3\alpha+1)(\alpha^2+3\alpha+3)\right)^{1/2}\right)/\left(\alpha^2+\alpha+1\right)$.

The third root, $\tilde{\tilde{q}}_i$, is greater than 1 for any $\alpha\in(0,1]$. From the concavity of *num* in $[0,1]^2$ and the values of the three roots, we have that for any $\alpha\in(0,1]$, *num* > 0 if $q_i<\tilde{q}_i$; *num* = 0 if $q_i=\tilde{q}_i$; and *num* < 0 if $q_i>\tilde{q}_i$.

We now have that if x_i is sufficiently small so that in equilibrium $q_i\in(0,\tilde{q}_i)$, then $d\left(y_{ig}^*+\hat{y}_{jg}+\hat{y}_{kg}\right)/dx_i>0$; and if x_i is sufficiently large so that in equilibrium $q_i\in(\tilde{q}_i,1)$, $d\left(y_{ig}^*+\hat{y}_{jg}+\hat{y}_{kg}\right)/dx_i<0$.

To complete the proof, we need to establish that if $x_i<\min\{x_j,x_k\}$, then $q_i\in(0,\tilde{q}_i)$; and if $x_i>\max\{x_j,x_k\}$, $q_i\in(\tilde{q}_i,1)$. From Lemma 1 (i.e., $x_j>x_k \Leftrightarrow q_j>q_k$) and the requirement that

$\alpha \in (0,1]$, we have that $x_j \geq x_k$. Hence, we need that if $x_i < x_k$ and $q_i < q_k$, then $q_i \in (0, \tilde{q}_i)$; and if $x_i > x_j$ and $q_i > q_j$, then $q_i \in (\tilde{q}_i, 1)$. We examine two cases.

***Case 1*:** University *i* is the best university, $x_i > \max\{x_j, x_k\}$.

If $x_i > \max\{x_j, x_k\}$, then $q_i > \max\{q_j, q_k\}$. Hence, we exaggerate the positive responses by the competitors. That is,

$$\frac{\partial \hat{y}_{jg}}{\partial x_j} = \frac{\partial \hat{y}_{jg}}{\partial y_{ig}} = \frac{q_j}{1-q_i} q_i > \frac{q_j}{1-q_j} q_i = \frac{\partial y^*_{jg}}{\partial x_j} = \frac{\partial y^*_{jg}}{\partial y_{ig}} > 0. \tag{A25}$$

By exaggerating $\partial y_{jg} / \partial x_j$ and $\partial y_{jg} / \partial y_{ig}$ without changing y^*_{jg}, the exaggeration has only a second-order (i.e., very small) effect on dy^*_{ig} / dx_i. Hence, if $x_i > \max\{x_j, x_k\}$ and $d(y^*_{ig} + \hat{y}_{jg} + \hat{y}_{kg}) / dx_i < 0$, then $d(y^*_{ig} + y^*_{jg} + y^*_{kg}) / dx_i < 0$. Figure A1 shows both $d(y^*_{ig} + \hat{y}_{jg} + \hat{y}_{kg}) / dx_i$ and $d(y^*_{ig} + y^*_{jg} + y^*_{kg}) / dx_i$ for the case in which $x_i > \max\{x_j, x_k\}$.

[Insert Figure A1 about here]

Using $1 = q_i + q_j + \alpha q_j$, we derive that $q_i > q_j$ (and by Lemma 1 $x_i > x_j$) if and only if $q_i > 1/(2+\alpha)$. Therefore, we need that if $q_i > 1/(2+\alpha)$, then $q_i \in (\tilde{q}_i, 1)$. Equivalently, we need that $1/(2+\alpha) > \tilde{q}_i$. Using (A24), we have

$$\begin{aligned} \frac{1}{2+\alpha} - \tilde{q}_i &= \frac{1}{2+\alpha} - \frac{2(1+\alpha)^2 - \left((3\alpha^2+3\alpha+1)(\alpha^2+3\alpha+3)\right)^{1/2}}{(\alpha^2+\alpha+1)} \\ &= \frac{(1+2\alpha)(\alpha^2+3\alpha+3) + (2+\alpha)\left((3\alpha^2+3\alpha+1)(\alpha^2+3\alpha+3)\right)^{1/2}}{(\alpha^2+\alpha+1)(2+\alpha)}. \end{aligned} \tag{A26}$$

Then, since the denominator of r.h.s. of the second line of (A24) is positive,

$$sign\left(\frac{1}{2+\alpha} - \tilde{q}_i\right) = sign\left(-(1+2\alpha)(\alpha^2+3\alpha+3) + (2+\alpha)\left((3\alpha^2+3\alpha+1)(\alpha^2+3\alpha+3)\right)^{1/2}\right) \tag{A27}$$

Evaluating the r.h.s. of (A27), we have

$$\left(-(1+2\alpha)(\alpha^2+3\alpha+3)+(2+\alpha)\left((3\alpha^2+3\alpha+1)(\alpha^2+3\alpha+3)\right)^{1/2}\right)>0$$

if and only if

$$\frac{\alpha^2+4\alpha+4}{4\alpha^2+4\alpha+1}>\frac{\alpha^2+3\alpha+3}{3\alpha+3\alpha+1}. \tag{A28}$$

We have that for any $\alpha\in(0,1)$, (A28) holds (with a strict equality if $\alpha=1$).

***Case 2*:** University *i* is the lowest ranked university, $x_i<\min\{x_j,x_k\}$.

If $x_i<\min\{x_j,x_k\}$, then $q_i<\min\{q_j,q_k\}$. Hence, we shade the positive response by the competitor. That is,

$$\frac{\partial\hat{y}_{jg}}{\partial x_j}=\frac{\partial\hat{y}_{jg}}{\partial y_{ig}}=\frac{q_j}{1-q_i}q_i<\frac{q_j}{1-q_j}q_i=\frac{\partial y^*_{jg}}{\partial x_j}=\frac{\partial y^*_{jg}}{\partial y_{ig}}>0. \tag{A29}$$

By shading these positive responses, we have that if $x_i<\min\{x_j,x_k\}$ and $d\left(y^*_{ig}+\hat{y}_{jg}+\hat{y}_{kg}\right)/dx_i>0$, then $d\left(y^*_{ig}+y^*_{jg}+y^*_{kg}\right)/dx_i>0$. Figure A2 shows both $d\left(y^*_{ig}+\hat{y}_{jg}+\hat{y}_{kg}\right)/dx_i$ and $d\left(y^*_{ig}+y^*_{jg}+y^*_{kg}\right)/dx_i$ for the case in which $x_i<\min\{x_j,x_k\}$.

[Insert Figure A2 about here]

Using $1=q_i+q_j+\alpha q_j$ and $q_k=\alpha q_j$, we derive that $q_i<q_k$ (and by Lemma 1 $x_i<x_k$) if and only if $q_i<\alpha/(1+2\alpha)$. Therefore, we need that if $q_i<\alpha/(1+2\alpha)$, then $q_i\in(0,\tilde{q}_i)$. Equivalently, we need that $\tilde{q}_i-\alpha/(1+2\alpha)>0$. Using (A24), we have

$$\begin{aligned}\tilde{q}_i-\frac{\alpha}{1+2\alpha}&=\frac{2(1+\alpha)^2-\left((3\alpha^2+3\alpha+1)(\alpha^2+3\alpha+3)\right)^{1/2}}{(\alpha^2+\alpha+1)}-\frac{\alpha}{1+2\alpha}\\&=\frac{(2+\alpha)(\alpha^2+3\alpha+1)-(1+2\alpha)\left((3\alpha^2+3\alpha+1)(\alpha^2+3\alpha+3)\right)^{1/2}}{(\alpha^2+\alpha+1)(1+2\alpha)}.\end{aligned} \tag{A30}$$

Then, since the denominator of r.h.s. of the second line of (A30) is positive,

$$sign\left(\tilde{q}_i - \frac{\alpha}{1+2\alpha}\right) = sign\left((2+\alpha)(\alpha^2+3\alpha+3) - (1+2\alpha)\left((3\alpha^2+3\alpha+1)(\alpha^2+3\alpha+3)\right)^{1/2}\right) \qquad \text{(A31)}$$

Evaluating the r.h.s. of (A31), we have that

$$\left((2+\alpha)(\alpha^2+3\alpha+3) - (1+2\alpha)\left((3\alpha^2+3\alpha+1)(\alpha^2+3\alpha+3)\right)^{1/2}\right) > 0 \qquad \text{(A32)}$$

if and only if (A28) holds.

Q.E.D.

Proof of Lemma 2

If $(x_i - x_j) = (\hat{x}_i - \hat{x}_j)$ for each $i, j \in \{1,2,3\}$, then from expression (2),

$$q_i(x_1 + y_{1g}, x_2 + y_{2g}, x_3 + y_{3g}) = q_i(\hat{x}_1 + y_{1g}, \hat{x}_2 + y_{2g}, \hat{x}_3 + y_{3g}). \qquad \text{(A35)}$$

For the two quality vectors, (x_1, x_2, x_3) and $(\hat{x}_1, \hat{x}_2, \hat{x}_3)$, the effect of y_{ig} on the expected scores is identical. That is, for each university *i* and for each candidate *g*:

$$\frac{\partial E[\sigma_i(x_1 + y_{1g}, x_2 + y_{2g}, x_3 + y_{3g})]}{\partial y_{ig}} = [w_p(v_i - v_{io}) - w_r y_{ig}]\frac{(1 - q_i(x_1 + y_{1g}, x_2 + y_{2g}, x_3 + y_{3g}))}{\mu} - w_r \qquad \text{(A36)}$$

$$= \frac{\partial E[\sigma_i(\hat{x}_1 + y_{1g}, \hat{x}_2 + y_{2g}, \hat{x}_3 + y_{3g})]}{\partial y_{ig}} = [w_p(v_i - v_{io}) - w_r y_{ig}]\frac{(1 - q_i(\hat{x}_1 + y_{1g}, \hat{x}_2 + y_{2g}, \hat{x}_3 + y_{3g}))}{\mu} - w_r.$$

Hence, with these identical derivatives, for each *i* and for each *g*, $y^*_{ig}(x_1, x_2, x_3) = y^*_{ig}(\hat{x}_1, \hat{x}_2, \hat{x}_3)$.

Q.E.D.

Proof of Theorem 2

Using (A9),

$$\frac{\partial y^*_{ig}}{\partial(v_{ig} - v_{io})} = w_p(1 - q_i) > 0. \qquad \text{(A37)}$$

Using (A9), for any $j, k \in \{1,2,3\}$,

$$\frac{\partial q_j}{\partial y_{kg}} = \frac{q_j q_k}{\mu} > 0 . \tag{A38}$$

The direct effect, as expressed in (A37), of an increase in $\left(v_{ig} - v_{io}\right)$ on y_{ig}^* is positive. All secondary effects, as expressed in (A38), of an increase in one university's merit aid offer on its competitors' merit aid offers is positive. Hence, for each $j \in \{1,2,3\}$, in equilibrium

$$\frac{\partial y_{jg}^*}{\partial\left(v_{ig} - \tilde{v}_{io}\right)} > 0 . \tag{A39}$$

Q.E.D.

Figure A1. Case: University i is the best university: The effect of an increase in university i's quality on the sum of the merit aid offers. If $x_i > \max\{x_j, x_k\}$, then

$$d\left(y_{ig}^* + y_{jg}^* + y_{kg}^*\right)/dx_i < d\left(y_{ig}^* + \hat{y}_{jg} + \hat{y}_{kg}\right)/dx_i < 0 .$$

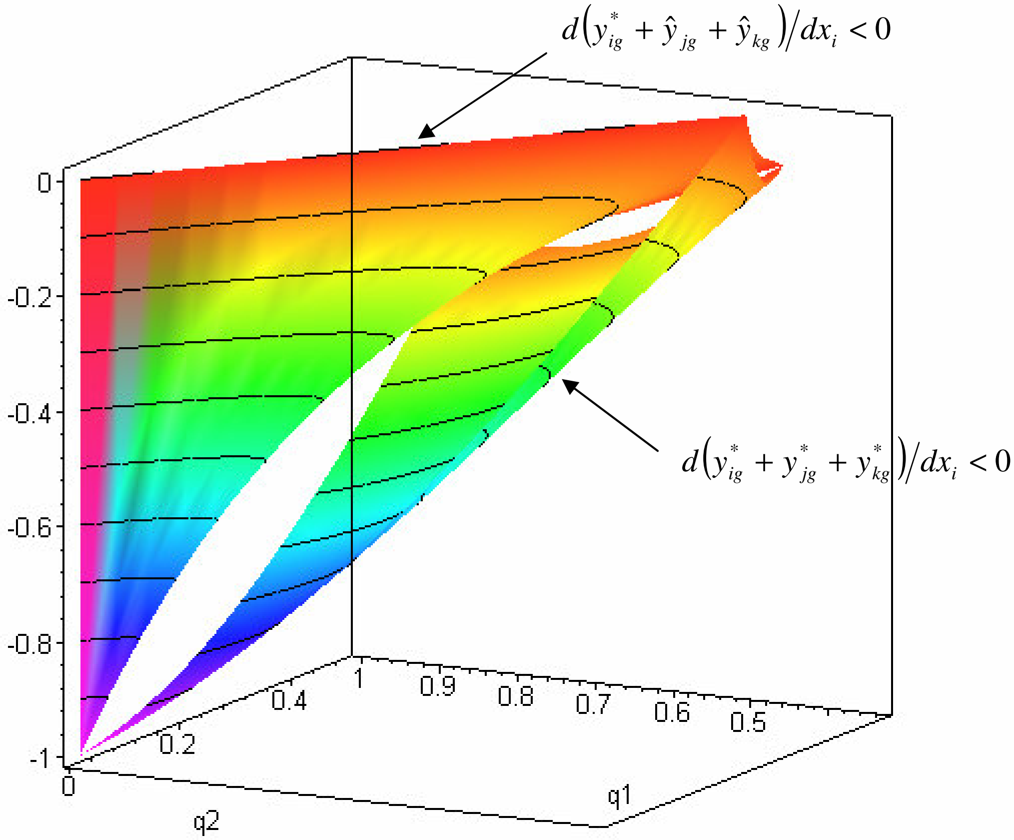


Figure A2. Case: University i is the worst university: The effect of an increase in university i's quality on the sum of the merit aid offers. If $x_i < \min\{x_j, x_k\}$, then

$$d\left(y_{ig}^{*} + y_{jg}^{*} + y_{kg}^{*}\right)/dx_i > d\left(y_{ig}^{*} + \hat{y}_{jg} + \hat{y}_{kg}\right)/dx_i > 0\,.$$